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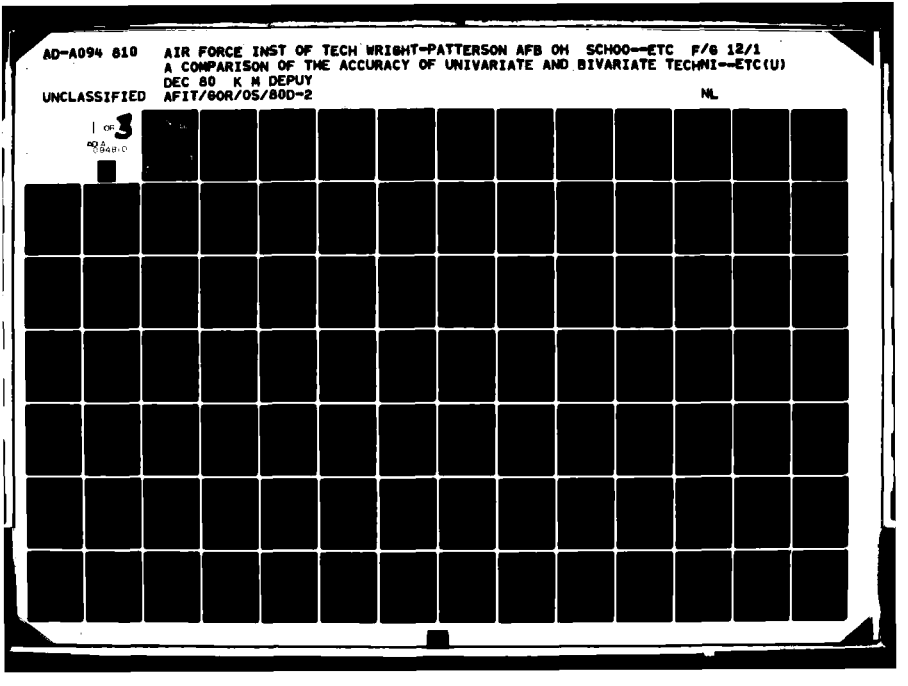

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LEVEL II

A COMPARISON OF THE ACCURACY OF UNIVARIATE AND BIVARIATE TECHNIQUES FOR FINDING THE LOWER CONFIDENCE LIMITS OF SYSTEM RELIABILITY

THESIS

AFIT/GOR/OS/80D-2 Kathleen M. DePuy
Captain USAF

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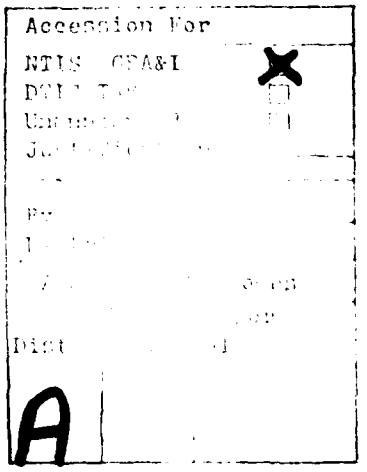
AFIT/GOR/OS/80D-2

A COMPARISON OF THE ACCURACY OF UNIVARIATE AND BIVARIATE TECHNIQUES FOR FINDING THE LOWER CONFIDENCE LIMITS OF SYSTEM RELIABILITY.

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science



by

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December 1980

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Preface

This thesis is a comparison of the accuracy of two Monte Carlo techniques of system reliability estimation previously developed at the Air Force Institute of Technology. In addition, two attempts at improving the accuracy of one of these techniques are made.

I wish to thank my thesis advisor, Dr. Jon R. Hobbs, for his suggestions, assistance, and patience during my study of this topic. I would also like to thank Dr. Albert H. Moore for suggesting this topic and for his initial guidance in starting this project. Thanks also goes to Dee Babiarz who typed this thesis.

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Abstract

The purpose of this thesis is to compare the accuracy of two Monte Carlo simulation techniques of finding lower system reliability confidence limits: the bivariate technique and the univariate technique. The actual results compared are the confidence interval coverages of the true system reliability associated with the confidence limits.

The bivariate technique is based upon the assumption that the maximum likelihood estimators of the component shape and scale parameters have an asymptotic normal distribution. The univariate technique uses the assumption that the component reliability estimates have a normal distribution. Two variations of the univariate technique are also examined. The first variation assumes that component reliability estimates follow a beta distribution instead of a normal distribution. The second variation replaces all perfect system reliability estimates with new, adjusted reliability values.

The results show that the bivariate technique is the most accurate technique if the true system reliability is believed to be below 0.95 in value. The univariate technique which is adjusted for perfect system reliability estimates is the most accurate technique if the true system reliability is believed to be at least 0.95 in value and if the component data used has a sample size of twenty or less.

The use of the beta distribution in place of a normal distribution for component reliability estimates in the univariate technique proved to be more accurate only at higher confidence levels and only more accurate than the original univariate technique.

A COMPARISON OF THE ACCURACY OF UNIVARIATE AND BIVARIATE TECHNIQUES FOR FINDING THE LOWER CONFIDENCE LIMITS OF SYSTEM RELIABILITY

I. Introduction

The military services have been seriously concerned with system reliability since World War II when the field failure of their equipment became severe. At the end of the war, the Department of Defense conducted reliability studies over a five year period (1945 to 1950). It found that electronic equipment used during Naval maneuvers only worked thirty percent of the time, that up to three-fourths of Army equipment was out of commission or under repair, and that equipment maintenance and repair costs for the Air Force were ten times the original cost of the equipment. As a result of these findings, the Department of Defense established an ad hoc committee on reliability in 1950, formalized the committee (AGREE--Advisory Group on the Reliability of Electronic Equipment) in 1952, and began publishing reliability specifications for its military electronic equipment in 1957 (Ref 27:1,12).

Although advances in technology since World War II have resulted in increased system reliability, the military services are still concerned with system reliability today because of the varied environments in which their systems

must function (the failure rate increases as the environment becomes more severe (Ref 27:2-3), the changed nature of conflict (there no longer is one weapon per soldier but systems of weapons (Ref 4:2-3), and the increased number of people affected by the weapons used in a conflict. Since the exact reliability of a system cannot be measured directly before it is put into field operation unless highly expensive testing is conducted, the reliability is estimated while the system is still being designed and built. The military services engage in reliability prediction not only to assure that the system will achieve its mission once it is in the field, but also to help pinpoint potential problems in the design before the system is built and to achieve an economical design that will minimize system life cycle costs (Ref 4:208,211).

Because of continuing budget constraints, reliability prediction will have to be made using less system testing and more efficient and effective methods of reliability estimation. As a result, the military services are exploring methods for predicting reliabilities of complex systems from system component test data. Since these predictions can vary considerably in accuracy, upper and lower limits are attached to the probable range of the predictions. When these limits are determined to a desired degree of confidence, they are called reliability confidence limits.

Background

A method for determining the lower confidence limit of the estimated system reliability was presented in 1960 by Donald Orkand. This proposed technique used Monte Carlo simulation to generate system reliability estimates using sample component failure data. He chose to concentrate on finding the lower confidence limit since this limit is the demonstrated system reliability. Orkand pointed out that the technique could be used even if sample sizes varied from component to component. At this point, no computer program for this technique had been developed and no techniques existed for determining adequate Monte Carlo sample sizes (Ref 23:4,6).

Since 1963, graduate students at the Air Force Institute of Technology have been investigating the use of Monte Carlo techniques for estimating system reliability. Their research has been performed under the direction of Dr. Albert H. Moore who implemented, improved, and broadened the application of this technique for obtaining system reliability confidence limits from component failure test data. Of particular interest are the research studies of Bernhoff, Levy, Lutton, Lannon, and Putz.

In 1963, Bernhoff demonstrated that a popular procedure of finding system reliability confidence limits was incorrect. The procedure was to find the component reliability confidence limits for a given confidence level, combine them

according to the probability laws of reliability, and use the resulting value as the system reliability confidence limit with the same confidence level as that of the components. Bernhoff found that system reliability confidence limits and their associated confidence intervals could be accurately determined from an empirical distribution of system reliability estimates. This empirical distribution is formed by combining the probability distribution functions of component reliability estimators. If all of the components fail independently of one another, are serially connected, and belong to the same family of failure distributions, the system reliability confidence limit can be found analytically; however, if the components are not all independent, are not serially connected, or if one or more component failure distributions belong to a different family, mathematical simulation--specifically, the Monte Carlo method--must be used to find the reliability confidence limit for the system because the analytical approach becomes impractical (Ref 3:3,36).

In 1964, Levy coded this Monte Carlo method into a computer program to find system reliability confidence limits at specified confidence levels. He showed that the Monte Carlo method can be used to accurately predict a system's reliability when the component failure patterns follow one or more of five different probability distribution functions: exponential, Weibull, gamma, normal, and log normal. This

coded technique required that the maximum likelihood estimator values of the distribution parameters be supplied by the user and only component reliability and system reliability estimates were generated by Monte Carlo simulation. Levy found that sample runs of 100 reliability values resulted in system reliability estimates that differed by less than one percent from the analytically calculated system reliability values (Ref 14:1,12,35-38). These findings were published by Levy and Moore in 1967 (Ref 15).

During that same year, Lutton used the asymptotic distribution of the parameter estimators of the Weibull (location parameter known), the gamma (location parameter known), and the logistic distributions to find system reliability confidence limits. In order to use Lutton's computer program, one still had to supply the maximum likelihood estimators of the parameter values of each component's life distribution as well as the elements of the variance-covariance matrix. At that time, asymptotic variances and covariances were tabled only for integer values of the shape parameters of the Weibull and gamma distributions. This, plus the number of Monte Carlo simulations performed and the size of the component samples, affected the accuracy of the confidence limits found for system reliability (Ref 17:2,16,28).

In 1972, Lannon developed a bivariate asymptotic technique based on Monte Carlo simulation to find system

reliability confidence limits. The failure times of each system component had a Weibull distribution with a known location parameter and unknown shape and scale parameters. He found that the accuracy of the method increased as the number of system reliability simulations increased and as the size of sample component failure times increased. Lannon's computer program included estimation of the maximum likelihood estimators of distribution parameters and the calculation of the variance-covariance matrix elements.

In 1979, Putz used a univariate asymptotic technique to generate lower confidence limits and the associated confidence interval coverage of the true system reliability. Using components whose failure times had a Weibull distribution with unknown scale and shape parameters, he ran Monte Carlo simulations of system reliability for components whose sample data sizes ranged from 10 to 100. He found that confidence interval coverage improved as the component sample size increased. He concluded that the univariate method works best when the component and system reliabilities are less than 0.9. When the reliabilities are greater than 0.9, large component test data sizes should be used in the estimation technique. Putz's computer program included an algorithm to find the maximum likelihood estimators of the Weibull distribution parameters.

In 1979, Moore, Harter, and Snead compared the accuracy of the univariate and bivariate methods of estimating system

reliability. They concluded that the bivariate method was more conservative, more accurate, and less sensitive to degradation due to high system reliability than the univariate method. However, the comparison of the two techniques was not based on each technique using the same component sample sizes, the same parameters for the Weibull distribution, the same number of simulation runs, and more than one system network (Ref 21:9,12). In order to obtain a more accurate comparison of these two techniques, this project was undertaken to create an experimental environment which varies only the techniques involved.

Problem Statement and Scope

The purpose of this study is to compare a univariate technique with a bivariate technique of predicting system reliability confidence limits. The univariate and bivariate techniques used were already developed by past thesis students, Putz and Lannon repsectively. Each of these techniques is applied to the same set of components, systems, and sample sizes so that only the techniques themselves vary. The comparison is based upon the five sample sizes, four systems, five components, seven confidence levels, and six hundred simulation runs used by Putz in his research so that Putz's results can be used to represent one case of the univariate technique.

Assumptions

The comparison of techniques has the following seven underlying assumptions:

1. Prior to the use of these reliability prediction techniques, all system components have been subjected to reliability tests. The resulting failure times for each component have a Weibull probability distribution. Thus one can estimate the scale, shape, and location parameters of each component failure distribution.

2. The location parameter is known or can be set equal to zero for all Weibull component distributions.

3. All system components fail independently of one another. This will simplify the calculation of system reliability.

4. Mission time is 100 hours, an arbitrary value.

5. Putz and Lannon's computer programs accurately simulate the basic univariate and bivariate techniques.

6. Putz's thesis results, i.e., his confidence intervals associated with the lower confidence limits, are valid and therefore can be used to represent one case of the univariate technique.

7. Gatliffe's technique of adjusting for zero component failures is valid. This adjustment is applied to the univariate cases in an attempt to improve the accuracy of the univariate technique.

Assumptions one through four are part of the univariate and bivariate models used by Putz (Ref 24:7) and Lannon (Ref 12:4).

General Approach

To accomplish the objective of comparing the accuracy of the univariate and bivariate techniques of finding system reliability confidence limits, the following specific cases of the techniques were tested and compared:

1. Putz's univariate approach which assumes that component reliability estimates are normally distributed (univariate--normal technique).

2. Putz's univariate approach which is modified to assume that component reliability estimates have a beta distribution (univariate--beta technique).

3. The above univariate-normal technique with the addition of Gatliffe's method to adjust for unrealistic estimates of perfect system reliability.

4. The bivariate asymptotic technique as outlined by Lannon in his research (bivariate technique).

Although both techniques calculate the lower confidence limits of system reliability estimates for specified confidence levels, the actual results compared are the confidence interval coverages of the true system reliability associated with the confidence limits.

Sequence of Presentation

This report begins in Section II with a review of the major concepts of reliability theory and a discussion of the use of the Weibull distribution function as a model of component failure times. The maximum likelihood estimation of component parameters and the general procedure of the Monte Carlo technique are then presented. This is followed by a discussion of finding system confidence limits and their associated confidence intervals which will be used later in the analysis of results. Then the theory behind the univariate and bivariate methods of generating a distribution of system reliability points is presented. Section II concludes with a discussion of adjusting these methods in the event of perfect system reliabilities. Section III outlines the system networks, component distributions, and the modifications made to the univariate and bivariate techniques. Section IV presents the results: the confidence intervals found from each technique and a comparison of them. The conclusions and recommendations in Section IV complete this thesis presentation.

II. Theory and Development of Techniques

The main concepts underlying the univariate and bivariate system reliability estimation techniques span several branches of mathematics including calculus, probability, statistics, and linear algebra. To prepare for the casual use of these mathematical principles in the discussion of the two Monte Carlo simulation techniques, background material is first presented on component and system reliability concepts, the Weibull component failure distribution function used in the two techniques, and the method of estimating parameters of the Weibull distribution. Following this is an outline of the general procedures used in estimating a system's reliability via Monte Carlo simulation. A brief review of confidence intervals, levels, and limits is also presented before liberally using these terms in the discussion of the two techniques. The results obtained by Lannon and Putz as well as the bivariate and univariate procedures themselves are then presented. This section ends with the logic Gatliffe used to replace perfect system reliability estimates with more realistic values in the calculation of lower confidence limits of a system's reliability.

Reliability of Systems and Components

Reliability is the probability that a device will operate satisfactorily under specified environmental conditions for a given period of time (Ref 4:28). Given a

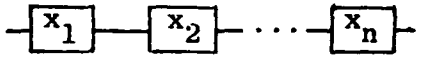
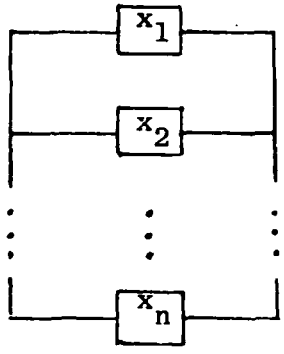
failure density function, $f(t)$, and a random failure time, T, of a system or a component, the probability of its failure as a function of time is $P(T \leq t) = F(t) = \int_0^t f(t)dt$. Therefore, the reliability of the system or component, $R(t)$, is the probability of success as a function of time, i.e., $R(t) = P(T>t) = 1-F(t)$ (Ref 27:180). Since reliability is a probability, its value is always a real number between 0 and 1.

In the field of engineering, there are five main ways to help achieve the desired reliability of a system. They are the use of (1) reliability prediction, (2) built-in test equipment, (3) debugging or burn-in, (4) testing to destruction, and (5) selection of system parts on the basis of special tests. Methods two through five are costly, and the last method has been found to contribute very little to the final system reliability (Ref 4:3). Thus, industry as well as the military has focused on the first method--reliability prediction.

Reliability estimation is the calculation of a system's reliability based on the reliability of its parts (Ref 4:3). Four ways to estimate reliability are guess work, extrapolation, mathematical calculation, and measurement. Because measurement is costly and guess work and extrapolation are the least accurate, mathematical calculation techniques are of high interest to anyone concerned with reliability. Mathematical calculation uses the laws of reliability

theory, data from past experiments, and functional diagrams showing the relationship of components in logical series or parallel to each other in the system (Ref 4:208-210). The reliability of any system can be found by reducing the system network to a combination of serial and parallel configurations and by using the reliability expressions for series and parallel component networks as shown in Figure 1. As the mathematical expressions indicate, the system's reliability is dependent upon each component's reliability. If the components are connected serially, the failure of any one of them will cause the system to fail. If the components are connected in parallel, the system will fail only if all components fail.

In most studies, all system components are assumed to fail independently of one another. This means that a component failure is due to some random event and that it does not occur as a result of the effects generated by other system component failures (Ref 4:85,296). Not only is this assumption made in order to use a simpler mathematical expression for calculating system reliability but also because system reliability is often defined to be a function of relatively independent critical components rather than a function of all components. For example, large Air Force systems are often designed using a system-subsystem structure. Each subsystem is further divided into parts. If the parts interact heavily, the reliability diagram of the system is

Configuration	Failures	Reliability Expression
Series	Dependent	$P(x_1)P(x_2\|x_1)P(x_3\|x_1x_2)\ldots P(x_n\|x_1x_2\ldots x_{n-1})$
	Independent	$\prod_{i=1}^{n} P(x_i)$
	Identical Components	ρ^n
Parallel	Dependent	$1-P(\bar{x}_1)P(\bar{x}_2\|\bar{x}_1)P(\bar{x}_3\|\bar{x}_1\bar{x}_2)\ldots P(\bar{x}_n\|\bar{x}_1\bar{x}_2\ldots \bar{x}_{n-1})$
	Independent	$1 - \prod_{i=1}^{n} P(\bar{x}_i)$
	Identical Units	$1 - (1-\rho)^n$

where x_i = component i, $P(x_i) = \rho_i$, $P(\bar{x}_i) = (1-\rho_i)$, and $0 \leq \rho_i \leq 1$

Fig 1. Reliability Laws (Ref 27:124,149)

complex and difficult to draw. In such cases, the Air Force identifies critical components, i.e., single components or groups of components for which one failure will lead to an inoperative or severely degraded system. If the definition of component failures results in little interaction among the critical component failures, then the system reliability may be written as the product of independent component success probabilities (Ref 27:140-141).

Because a system's reliability is a function of its components' reliabilities, researchers have concentrated on finding component reliabilities. No general theory exists to tell technologists how to calculate the reliability of a single part or component using the basic physics of failure of parts, i.e., the knowledge of how the part was made including the materials and the temperature, pressure, humidity, and voltage conditions used. For this reason, component reliabilities must also be estimated. Reliability estimates use experimental failure data (i.e., the failure times of items placed on a life test) and/or the mean-time-between-failure data from systems already in operation (i.e., the operating hours of replaced parts in equipment already in field use) (Ref 27:159-160). In both cases, a technologist uses the failure data to plot a histogram. He then chooses an appropriate failure distribution model based on the resulting histogram and uses the model to estimate the parameters of the distribution (Ref 27:441). This thesis

assumes the distribution of the components has already been found and is concerned with the first type of failure data--experimental failure data.

To collect experimental failure data for a certain type of component, a life test of the component is conducted. One consideration is the component sample size, i.e., how many identical components to test. Life testing is time-consuming; it may take years before a failure occurs. The smaller the component sample size, the more time is needed for testing in order to collect data. If on the other hand, only two or three components are tested for a short time, too little data will exist and no accurate quantitative conclusions can be drawn about the component and system's reliabilities regardless of how powerful a reliability calculation technique is used. This implies that one should increase the sample size used. However, life testing is also costly; the cost is due to the labor required to set up a controlled environment for the test and sometimes due to the price of the components being tested (an inexpensive resistor or an expensive gyro). Cost also increases because the data obtained will only apply for the component if it is used in that particular environment; another test must be conducted to see how the component will perform in a different environment. These cost considerations imply that it would be desirable to reduce the sample size used. Past work in this area has shown that in order to get the most

information from a testing situation, 20 to 100 components should be tested (Ref 27:441,457). Thus, this thesis project used component sample sizes that usually are in this recommended range.

Weibull Distribution

As stated in Section I, this research project assumes that the failure times of all system components have a Weibull density function. A failure density function is the "measure of overall speed [frequency] at which failures are occurring" (Ref 27:161). It is also a mathematical description of the length of life of material, a structure, or a device. The particular family of density functions chosen to model an item depends upon the relationship between the function's failure rate and the item whose life is being modeled. The exponential, gamma, Weibull, modified extreme value, truncated normal, and log normal distributions have all been used to model the fatigue failure of materials and the life length of electronic and mechanical components (Ref 2:9,12).

In 1939, a professor at the Royal Institute of Technology in Sweden proposed the use of a new distribution function to describe the life length of materials. This professor, Waloddi Weibull, published an article in 1951 in which he justified the use of this new function. He stated that for the class of failure distributions of the form

$F(x) = 1 - e^{-\sigma(x)}$, the only condition $\sigma(x)$ must satisfy is to be positive, nondecreasing, and vanishing at a value x_u which is not necessarily equal to zero. The simplest function that satisfies these three conditions is $\sigma(x) = \frac{(x-x_u)^m}{x_o}$. Thus, the resulting failure distribution function, now known as the Weibull distribution, is $F(x) = 1 - \exp\left(\frac{(x-x_u)^m}{x_o}\right)$ (Ref 30:293).

Like other distribution functions, the Weibull distribution function had no concrete theoretical foundation but was based upon empirical evidence. At that time, empirical evidence showed that the Weibull distribution often modeled observed failure data better than other known distribution functions. In his article, Professor Weibull give seven specific examples, with supporting data, to demonstrate the wide applicability of this function. The examples included the yield strength of steel, the fiber strength of cotton, the length of cyrtoideae, the height of men, and the breadth of beans. He also argued that until a better function is found, this empirically tested function should be used since it is the simplest function for that class of distributions (Ref 30:293-297).

Historically, the rival function of the Weibull distribution has been the exponential density function. Its popularity is due to both publicized empirical evidence and to its simple form. In 1951, Epstein and Sobel published

many important papers on life-testing which led to widespread use of the exponential distribution in life-testing research. In 1952, D. J. Davis published a paper in which he presented the results of goodness-of-fit tests for various failure distributions for given sets of failure data. The data was found to best fit the exponential distribution. Although Professor Weibull also published his findings on using the Weibull distribution to model fatigue failure in 1951, it was not until 1959 that real interest in the Weibull function grew. In that year, Zelen and Dannemiller published a paper showing that many life test procedures based on the exponential function were not robust. Before 1959, the Weibull distribution had been used occasionally to model, e.g., ball bearing failures (Lieblein and Zelen, 1956) and vacuum tube failures (J. H. Kao, 1958) (Ref 2:3,4,16).

The exponential distribution is often chosen to model failures because it has many desirable mathematical properties. For example, its use involves the simple addition of failure rates and a simple form into which design data can be compiled. However, this distribution has limited applicability because of a property it has that leads to an unrealistic representation of many items. This property is that an item being used never ages; no deterioration in performance occurs before it fails. Therefore, if an item has not failed up to a time t, the probability distribution of its future life length $(T-t)$ is the same as if the item

were new and just beginning to be used at time t . In 1957, W. Feller showed that the exponential distribution is the only continuous distribution with this constant failure rate property (Ref 2:4,13,14,18).

Because most materials, structures, and devices wear out with time, distributions with increasing failure rates are of interest. Distributions with decreasing failure rates are also of interest because of empirical observations that show decreasing failures for a process. Two examples of decreasing failure distributions are the work hardening of certain materials and the debugging of complex systems (Ref 2:18-22). Although the exponential density function has a constant failure rate, it does realistically represent some items such as an electric fuse which cannot partially melt. Often the exponential function is also used to represent component failure times because of the mid-life description of a "typical" component. Early in the life of a typical component, many failures are due to initial weaknesses or defects such as poor insulation or bad assembly in the system. Late in the life of the component, many failures are due to the deterioration of the component. However, in the component's mid-life, fewer failures exist. The ones that do exist are often due to environmental stresses that exceed the component's design strength. It is difficult to predict the environmental stress amplitudes or the component strengths as deterministic functions of time (Ref 27:170-171).

Enough comparative analysis has not been done on different types of components to label any class of components with a particular failure distribution. For example, all electronic components do not have an exponential failure rate; technologists only know that many do (Ref 27:172). Although the exponential distribution has been a rival function of the Weibull distribution, it is in fact a special case of the Weibull distribution. Thus, using the Weibull family of distribution functions to model component failures allows the modeling of increasing, decreasing, or constant failure rates as discussed next.

The Weibull probability distribution function of a random variable T has three parameters and is described mathematically as:

$$f(t) = \begin{cases} \frac{k(t-c)^{k-1}}{\theta^k} e^{-(\frac{t-c}{\theta})^k} & , \theta , k>0 \text{ and } t \geq c \\ 0 & , \text{elsewhere} \end{cases} \quad (1)$$

The mean value and variance of this distribution are:

$$E(t) = c + \theta\Gamma(1+\frac{1}{k}) \quad (2)$$

$$Var(t) = \theta^2[\Gamma(1+\frac{2}{k}) - \Gamma^2(1+\frac{1}{k})] \quad (3)$$

The location parameter, c , is the point of origin of the distribution, i.e., the point in time, t , at which failures begin to occur. The scale parameter, θ , represents

the characteristic life or spread of the function about its mean. The shape parameter, k , represents the failure rate; it can be increasing (k>1), decreasing (k<1), or constant (k=1) over time (Ref 2:13; 24:3; and 18:184-185). When k=1 , the Weibull distribution is the exponential function; when k=2 , it is the Rayleigh distribution; and, when k=3.5 , it approximates the normal distribution (Ref 24:3 and 18:128).

The Weibull distribution can be used to model an item in which a large number of weaknesses exist and whose failure depends on its worst weakness if the distribution of weaknesses is of the proper form. The Weibull distribution has been used to analyze the failures in electronic components, semiconductor devices, photoconductive cells, motors, and capacitors. It has been used in the study of biological organisms, psychological test situations, breaking strength and fatigue in textiles, corrosion resistance, leakage failure of dry batteries, return of goods after shipment, marketing life expectancy of drugs, number of downtimes per shift, and particle-size data (Ref 18:185).

If a component has failure times that are patterned according to the Weibull distribution, its reliability can be found using the general definition of reliability given at the beginning of Section II. Thus, the component reliability is

$$R(t) = \exp(-(\frac{t-c}{\theta})^k) \qquad (4)$$

This equation indicates that in order to obtain a value for the component reliability $R(t)$, values for t, c, θ, and k are needed. Since only component failure times, t, will be available, the values for c, θ, and k will have to be supplied if known and estimated if unknown. Use of estimated parameters will make the resulting value of Eq (4) an estimate also.

Maximum Likelihood Estimation of Component Parameters

In general, there are three mathematical techniques used to estimate the parameters of a distribution: (1) method of least-squares; (2) method of moments; and (3) method of maximum likelihood. The maximum likelihood technique is the most flexible and most powerful of the three methods (Ref 27:464). For this reason, it has been used to estimate distribution parameters by AFIT students conducting research into the Monte Carlo techniques for estimating system reliability. In particular, it has been used by Lannon and Putz in their separate work with the Weibull distribution. In addition, other researchers have used it in connection with the Weibull distribution. For example, in 1969, Darrel Thoman, Lee Bain, and Charles Antle showed that the maximum likelihood estimators of the three parameters of the Weibull distribution are better (less biased and

possess smaller variances) than other estimators of the parameters (Ref 28:457-458).

The maximum likelihood estimates for the shape and scale parameters (assuming the location parameter is known or zero) of the Weibull distribution are found in the standard manner by finding the first partial derivatives of the natural logarithm of the likelihood function with respect to each parameter, setting each of these derivatives equal to zero, and solving the resulting equations for the two parameters. The actual calculations and maximum likelihood estimates are as follows:

$$L = f(t_1)f(t_2)\ldots f(t_n)$$

$$L = \frac{k(t_1-c)^{k-1}}{\theta^k} e^{-(\frac{t_1-c}{\theta})^k} \frac{k(t_2-c)^{k-1}}{\theta^k} e^{-(\frac{t_2-c}{\theta})^k} \ldots \frac{k(t_n-c)^{k-1}}{\theta^k} e^{-(\frac{t_n-c}{\theta})^k}$$

$$L = \frac{k^n}{\theta^{kn}} \left(\prod_{i=1}^{n} (t_i-c)^{k-1} \right) e^{-\sum_{i=1}^{n} (\frac{t_i-c}{\theta})^k} \tag{5}$$

$$\ln L = n \ln \frac{k}{\theta^k} + \ln\left(\prod_{i=1}^{n} (t_i-c)^{k-1} \right) - (\frac{t_1-c}{\theta})^k - \ldots - (\frac{t_n-c}{\theta})^k$$

$$\ln L = n\ln k - nk\ln\theta + (k-1)\ln\left(\prod_{i=1}^{n}(t_i-c)\right) - \frac{1}{\theta^k}\left(\sum_{i=1}^{n}(t_i-c)^k\right) \quad (6)$$

$$\frac{\partial \ln L}{\partial \theta} = -\frac{nk}{\theta} + \frac{k}{\theta^{k+1}}\left(\sum_{i=1}^{n}(t_i-c)^k\right)$$

$$\frac{nk}{\hat{\theta}} = \frac{k}{\hat{\theta}^{k+1}}\left(\sum_{i=1}^{n}(t_i-c)^k\right)$$

$$\hat{\theta}^k = \frac{1}{n}\left(\sum_{i=1}^{n}(t_i-c)^k\right)$$

$$\hat{\theta} = \left(\frac{1}{n}\sum_{i=1}^{n}(t_i-c)^k\right)^{1/k} \quad (7)$$

$$\frac{\partial \ln L}{\partial k} = \frac{n}{k} - n\ln\theta + \sum_{i=1}^{n}\ln(t_i-c) - \sum_{i=1}^{n}\left(\left(\frac{t_i-c}{\theta}\right)^k \ln\left(\frac{t_i-c}{\theta}\right)\right)$$

$$\frac{1}{\hat{k}} = \ln\theta - \frac{1}{n}\sum_{i=1}^{n}\ln(t_i-c) + \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{t_i-c}{\theta}\right)^{\hat{k}} \ln\left(\frac{t_i-c}{\theta}\right)\right) \quad (8)$$

Because of the complexity of the equation defining the shape parameter estimate, the actual values of the parameters must be found simultaneously in an iterative manner rather than by a single simultaneous solution by direct substitution. Eqs (7) and (8) define the maximum likelihood estimates of

the shape and scale parameters when complete component test data is being used, i.e., when a life test is run until all components being tested fail and t_i represents the time at which the ith component failed (Ref 22:171).

In order to save time and cost during component testing, censored test data is often gathered rather than complete test data. Censored test data is gathered before all components fail; it consists of the failure times of the components that did fail plus the running times of the components that did not fail. There are two types of censored data, the type depending upon the manner of testing. Type I censoring of data is performed by running each component a prespecified time unless it fails sooner. Thus, the censoring times are fixed, and the number of failures is random. Type II censoring of data consists of simultaneously testing all components until a prespecified number of them fail. Thus, the number of failures is fixed and the common censoring time is random (Ref 22:171). When censored data is used, the maximum likelihood estimates of the parameters of the Weibull distribution are found using only the data for the components that failed. Thus, the following altered log-likelihood function (Ref 11:640) must be used:

$$L_r = \ln(n!) - \ln((n-m)!) - \ln(r!) + (m-r)(\ln k - k\ln\theta)$$

$$+ (k-1)\sum_{i=r+1}^{m} \ln(t_i - c) - \sum_{i=r+1}^{m} \left(\frac{t_i - c}{\theta}\right)^k$$

$$- (n-m)\left(\frac{t_m-c}{\theta}\right)^k + r\ln\left(1-\exp\left(-\left(\frac{t_{r+1}-c}{\theta}\right)^k\right)\right) \qquad (9)$$

where

n = total number of components tested

$m-r$ = number of components that failed

$n-m$ = number of components censored from above once the failure times are ordered

r = number of components censored from below once the failure times are ordered.

m = number of failure times in simulated life tests of n components $(m \leq n)$

t_i = ordered component failure times where i ranges from $r+1$ to m

The maximum likelihood equations are also found in the standard manner by setting the first partial derivatives of L_r, with respect to the parameters θ and k, equal to zero. The resulting equations for the shape and scale parameters (Ref 11:641) are:

$$\frac{\partial L_r}{\partial \theta} = \frac{-k(m-r)}{\theta} + k\sum_{i=r+1}^{m}\frac{(t_i-c)^k}{\theta^{k+1}} + \frac{k(n-m)(t_m-c)^k}{\theta^{k+1}}$$

$$- kr(t_{r+1}-c)^k\exp\left(-\left(\frac{t_{r+1}-c}{\theta}\right)^k\right)/\theta^{k+1}\left\{1-\exp\left(-\left(\frac{t_{r+1}-c}{\theta}\right)^k\right)\right\} \qquad (10)$$

$$\frac{\partial L_r}{\partial k} = (m-r)\left(\frac{1}{k} - \ln\theta\right) + \sum_{i=r+1}^{m}\ln(t_i-c)$$

$$- \sum_{i=r+1}^{m}\left(\left(\frac{t_i-c}{\theta}\right)^k\ln\left(\frac{t_i-c}{\theta}\right)\right) - (n-m)\left(\frac{t_m-c}{\theta}\right)^k\ln\left(\frac{t_m-c}{\theta}\right)$$

$$+ r(t_{r+1}-c)^k \ln\left(\frac{t_{r+1}-c}{\theta}\right)\exp\left(-\left(\frac{t_{r+1}-c}{\theta}\right)^k\right)/\theta^k\{1$$

$$- \exp\left(-\left(\frac{t_{r+1}-c}{\theta}\right)^k\right)\} \quad (11)$$

The corresponding equations for the complete test data situation are a special case of these equations. That is, if no component data is censored, then $r=0$ and $m=n$ and the above equations collapse to form Eqs (7) and (8).

Eqs (7) and (8) for the complete test data case are used in developing the univariate technique for estimating system reliability, and Eqs (10) and (11) for the censored test data case are used in developing the bivariate technique for estimating system reliability. Eqs (10) and (11) are used in the latter case to achieve flexibility, i.e., to enable the bivariate technique to handle either type of test data. Even though Eqs (10) and (11) are used in the bivariate technique, they will in essence collapse and become Eqs (7) and (8) for this thesis application.

Monte Carlo Techniques of Generating System Reliabilities

The general procedure followed in any Monte Carlo technique of estimating the lower confidence limit of system reliability consists of steps similar to the seven steps listed below:

1. For each component, generate a sample of component failure times using the component's distribution, $f(t)$, and the associated true parameters.

2. For each component, use the simulated component failure times to find the maximum likelihood estimates of the unknown parameters of the component's distribution.

3. For each component, generate a point estimate of the component reliability by substituting the known parameter values and the maximum likelihood estimates of the unknown parameters into the component reliability equation $R(t) = 1 - F(t)$. Go to Step 5.

4. For each component, generate a point estimate of the component reliability by substituting the known parameter values and new estimates of the unknown parameters into the component reliability equation $R(t) = 1 - F(t)$. Generate these new parameter estimates by using the maximum likelihood estimates and their assumed distribution.

5. Calculate a system reliability point estimate by substituting the component reliability point estimates into the reliability expression for the system network.

6. Repeat Steps 4 and 5 until the desired number of system reliability estimates is obtained.

7. Order the system reliability point estimates by increasing magnitude. The ordered points form a step cumulative distribution of system reliability estimates with each step equal to $1/n$ where n is the total number of system reliability estimates found. Find the $100(1-\alpha)$ percent lower confidence limit, where α is the desired significance level. It will be the value of the system

reliability point estimate that corresponds to the lower limit percentage point. For example, the lower confidence limit for a 95 percent confidence level is the point estimate at the 5 percent point of the ordered values (Ref 3:44).

In order to check the accuracy of the particular Monte Carlo estimation technique, three extra steps are added to the process. They are:

8. Calculate the true system reliability using the true component reliabilities plus the system reliability expression for the network. The true reliability of each component is found by substituting the component's known parameter values into its known distribution. The estimated lower confidence limit found in Step 7 is compared to the true system reliability to see if the $100(1-\alpha)$ percent confidence interval contains the true system reliability.

9. Repeat Steps 1 to 8 until the desired number of Monte Carlo simulations is reached.

10. Find the percentage of runs in which the confidence intervals cover the true system reliability. This percentage should be close to $100(1-\alpha)$ percent if the technique is accurate.

Once such a Monte Carlo technique is found to be within a desired accuracy, it can be used to find the lower confidence limit of an actual system's reliability. However, each component's distribution must be found using the actual component test data via some other method prior to using

this technique. Once each component's distribution is known, the actual component failure data from component testing would be input into Step 2 to estimate the unknown parameters of the component's distribution. One can then find the lower confidence limit of system reliability for a given significance level using Steps 3 through 7.

Confidence Levels, Limits and Intervals

Throughout the presentation of Monte Carlo techniques, three "confidence" terms are used in connection with the distribution of the estimated system reliability points: confidence level, confidence limit, and confidence interval. The confidence level, or confidence coefficient, is the degree that a given hypothesis is true within a specified limit(s). It can also be thought of as the number of times out of a given number of trials that an event is expected to occur (Ref 4:96,297). A confidence level is usually associated with an estimate in order to attach a degree of certainty to the precision of the estimate. The confidence level may be expressed as a probability $1-\alpha$ where α is a given significance level, or it may be expressed as a percentage $100(1-\alpha)$.

There are two confidence limits: upper and lower. The upper and lower confidence limits are the maximum and minimum limits respectively that define the range within which observations can be expected to occur with a specified

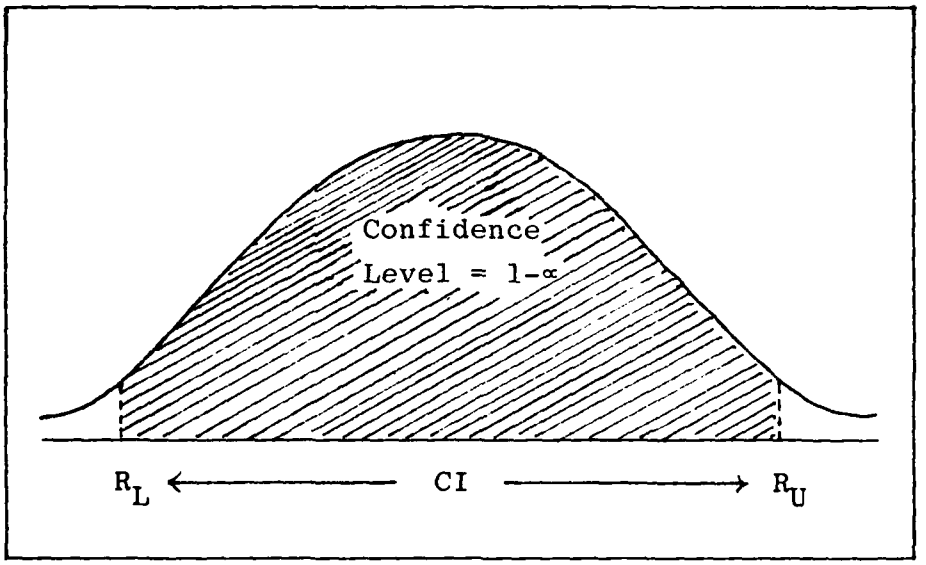


Fig 2. Confidence Concepts

degree of certainty (confidence level) (Ref 4:297). The range defined by the upper and/or lower confidence limits is called the confidence interval (Ref 19:274). Figure 2 shows a distribution of a random variable R plus a two-sided confidence interval (CI) for R defined by the upper and lower confidence limits R_U and R_L respectively, i.e., $CI = (R_L, R_U)$. The confidence level $1-\alpha$ is expressed as $P(R_L<R<R_U) = 1-\alpha$ and is the area under the curve enclosed by R_L and R_U .

In this thesis, one-sided confidence intervals are of interest since the estimation techniques will be finding only the lower confidence limit of system reliability. Thus

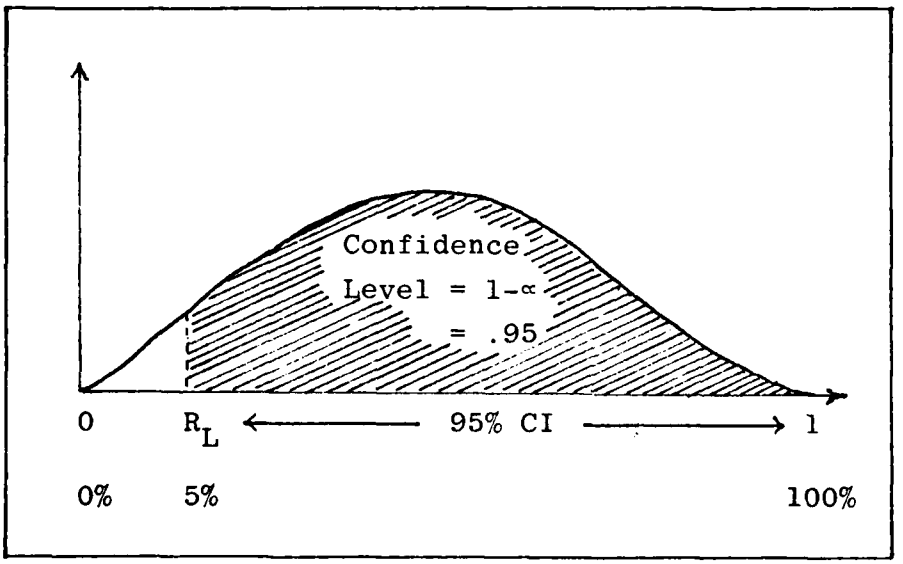


Fig 3. One-Sided Confidence Interval

for a given confidence level $1-\alpha$, the Monte Carlo techniques will estimate the lower confidence limit R_L of system reliability R and then find $P(R \geq R_L)$, i.e., $1-\alpha^*$ such that $P(R \geq R_L) = 1-\alpha^*$. If the technique is accurate, $1-\alpha^*$ will be close in value to $1-\alpha$. The implied confidence interval is $(R_L, 1)$ where 1 rather than ∞ is the upper bound of the interval because that is the maximum possible value of reliability. This one-sided confidence interval is illustrated in Figure 3 for a 95 percent confidence level.

Bivariate Technique

In 1972, Robert Lannon developed a bivariate Monte Carlo technique to estimate the lower confidence limit of a system's reliability when the component failure times have a Weibull distribution. He assumed that one parameter of this distribution, namely the location parameter, was known or equal to zero. The technique is called bivariate (two variables) not because the other two parameters of the Weibull distribution are unknown and must be estimated but because the component reliability estimate $\hat{R}(t)$ is a function of the estimates of these two unknown parameters.

Lannon's bivariate method parallels the general seven step Monte Carlo procedure for finding the system reliability confidence limits for a given confidence level. To use the bivariate technique, perform the following eight steps (Ref 12:7,8,15,51):

1. Generate random failure times for each system component using the known parameter values and the Weibull distribution function.

2. For each component, compute the maximum likelihood estimates of the component's shape and scale parameters.

3. Find the variance-covariance matrix of the asymptotic normal distribution of the shape and scale parameters for each component.

4. Generate a sample estimate of the component's shape parameter and a sample estimate of the component's

scale parameter using the maximum likelihood estimates as means of the bivariate normal distribution and the variance-covariance matrix elements to find the standard deviations of the sample estimates.

5. For each system component, calculate a point estimate of the component reliability by inserting the known location parameter and the sample estimates of the shape and scale parameters found in Step 4 into the reliability equation for the Weibull density function, i.e., Eq (4).

6. Calculate a point estimate of the system reliability by inserting the component reliability point estimates into the system reliability equation. The system reliability equation is found by using probability laws for the reliability of independent components based on the system design.

7. Repeat Steps 4 to 6 until the desired number of system reliability estimates is obtained.

8. Order the system reliability points in increasing magnitude to form the cumulative system reliability distribution. Find the desired confidence limit for a given confidence level $100(1-\alpha)$ by selecting the $100(\alpha)$ percent ordered sample system reliability point.

The first four of these eight steps make this Monte Carlo technique bivariate and for use with components whose failure times follow a Weibull distribution. Because of this, the theory behind each of these four steps is presented next.

In step one, it is assumed that the number of sample failure times is determined a priori and may vary from one component to the next. The inverse relation of the Weibull cumulative distribution function is used to generate the component failure times. The Weibull cumulative distribution function is

$$F(t) = 1 - \exp[-(\frac{t-c}{\theta})^k] \qquad (12)$$

Setting this function equal to variable Y, the inverse relation can be found and is

$$t = \theta(-\ln(1-Y))^{1/k} + c \qquad (13)$$

Because $F(t)$, and thus Y, belongs to the interval $[0.0, 1.0)$, pseudo-random numbers from a uniform distribution are generated for Y. Using the known values of the component parameters θ, k, and c, random Weibull values, t, are calculated for each component (Ref 12:7,51). The individual component failure times are ordered and used by a parameter estimation routine in step two.

The parameter estimation routine (PARES) computes the maximum likelihood estimates of the component shape and scale parameters. The actual subroutine used is an iterative procedure developed by Harter and Moore and can be used to estimate all three parameters of the Weibull distribution for complete or censored samples (Ref 11). The procedure uses the natural logarithm of the likelihood function

of all ordered failure times remaining after the sample is censored. That is, the procedure uses Eq (9). The maximum likelihood equations for each of the three parameters are found by taking the partial derivatives of L_r with respect to each parameter. These three equations are set equal to zero and are solved simultaneously via an iterative procedure which uses the actual parameter values as the initial estimates for the parameters, zero as the initial estimate for r, and the rule of false position (i.e., iterative linear interpolation) to find the maximum likelihood estimates. The three parameters are estimated iteratively in the following order: θ, k, and then c (Ref 11:641).

Linear interpolation is used to find the value of the parameter being estimated at any given step that satisfies the appropriate likelihood equation into which the latest estimates or known values of the other two parameters have been substituted. Positive values of $\hat{\theta}$ and $\hat{k}$ can always be found this way. If no value in the interval $(0,t_1)$ exists to satisfy the likelihood equation for c, then the likelihood function in this interval is either decreasing monotonically (and thus $\hat{c}=0$) or increasing monotonically (and thus $\hat{c}=t$, when $k\leq 1$) and this iterative estimation procedure cannot continue unless the procedure is modified. Harter and Moore state that a possible modification is to censor the smallest ordered failure time plus any other failure times equal to it. The variable r is set equal

to the total number of censored failure times. The iterative procedure can then continue until the results of the successive steps agree to within an assigned tolerance value (Ref 11:641). No modification to this iterative routine is needed in the bivariate method. Only the shape and scale parameter estimates generated in this iterative routine are used since the location parameter is assumed to be known in the bivariate procedure.

The next step in the bivariate technique is to find the variance-covariance matrix of the asymptotic normal distribution of the shape and scale parameters. Maximum likelihood equations can be used to approximate the values of the variances and covariances for two or more estimated distribution parameters. The general procedure is to calculate the second partial derivatives of the natural logarithm of the likelihood function L and form a matrix S. If any of the second partial derivatives have involved quantities such as a summation, the quantities should be replaced by their expected values. In the case of two parameters, θ and k, the matrix is

$$S = \begin{bmatrix} L_{\theta\theta} & L_{\theta k} \\ L_{\theta k} & L_{kk} \end{bmatrix} \qquad (14)$$

The variance-covariance matrix V of $\hat{\theta}$ and $\hat{k}$ will be the negative inverse matrix of S, i.e.,

$$V = -S^{-1} = \begin{bmatrix} \mathrm{Var}(\hat{\theta}) & \mathrm{Cov}(\hat{\theta},\hat{k}) \\ \mathrm{Cov}(\hat{\theta},\hat{k}) & \mathrm{Var}(\hat{k}) \end{bmatrix} \qquad (15)$$

These values in the variance-covariance matrix are valid only for large component sample sizes. For the Weibull distribution, an unknown amount of error exists in these values due to estimator bias. However, this bias disappears as the sample size increases. That is, a minimum attainable value for the variance of an estimator exists; the variances of the maximum likelihood estimators approach this minimum value as n increases toward infinity. The result of using these values when the component sample size is small is to underestimate the variances (Ref 16:174-177).

In 1967, Harter and Moore showed that the shape and scale estimators of the Weibull distribution are asymptotically unbiased, asymptotically of minimum variance, and asymptotically bivariate normal (Ref 10:561). They also discussed how to find the maximum likelihood information matrix and the variance-covariance matrix of the asymptotic normal distribution of the shape and scale parameters. The procedure begins with the natural logarithm of the likelihood function given in Eq (9). If the sample is complete (not censored), the elements of the information matrix are the limits as n approaches infinity of the negatives of the expected values of the second partial derivatives of the likelihood function with respect to each of the parameters

(Ref 10:558-559). If the sample is censored, these limits mentioned do not exist; the elements of the information matrix are the limits in probability of the conditional expectations (Ref 10:561).

The elements of the information matrix for the shape and scale parameter estimates are:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{k_{11}n}{\theta^2} & \frac{k_{12}n}{\theta} \\ \frac{k_{21}n}{\theta} & nk_{22} \end{bmatrix} \qquad (16)$$

where the values of k_{ij} are given by Harter and Moore as:

$$k_{11} = \lim_{n\to\infty} \text{pr } E\left[-\left(\frac{\theta^2}{n}\right)\left(\frac{\partial^2 L}{\partial\theta^2}\right)\right]$$

$$= -kp + k(k+1)\left[\Gamma(2;\hat{z}_m^k) - \Gamma(2;\hat{z}_{r+1}^k)\right] + k(k+1)q_2\,\hat{z}_m^k$$

$$+ \frac{\hat{z}_{r+1}f(\hat{z}_{r+1})[k\hat{z}_{r+1} - (k+1)q_1]}{q_1} \qquad (17)$$

$$k_{12} = k_{21} = \lim_{n\to\infty} \text{pr } E\left[-\left(\frac{\theta}{n}\right)\left(\frac{\partial^2 L}{\partial\theta\,\partial k}\right)\right]$$

$$= p - \left[\Gamma'(2;\hat{z}_m^k) - \Gamma'(2;\hat{z}_{r+1}^k)\right] - \left[\Gamma(2;\hat{z}_m^k)\right.$$

$$- \Gamma(2;\hat{z}_{r+1}^k)] - q_2\hat{z}_m^k(k\ln\hat{z}_m + 1) - \frac{\hat{z}_{r+1}f(\hat{z}_{r+1})}{kq_1}$$

$$\{k\hat{z}_{r+1}^k\ln\hat{z}_{r+1} - (k\ln\hat{z}_{r+1} + 1)q_1\} \quad (18)$$

$$k_{22} = \lim_{n\to\infty} pr\ E\ [-(\frac{1}{n})(\frac{\partial^2 L}{\partial^2 k})] = \frac{P}{k^2}$$

$$+ \frac{[\Gamma''(2;\hat{z}_m^k) - \Gamma''(2;\hat{z}_{r+1})]}{k^2} + q_2\hat{z}_m^k\ln^2\hat{z}_m$$

$$+ \frac{\hat{z}_{r+1}f(\hat{z}_{r+1})}{kq_1}\{\ln^2\hat{z}_{r+1}[\hat{z}_{r+1}^k - q_1]\} \quad (19)$$

where $z_i = \frac{(t_i-c)}{\theta}$, $p = 1-q_1-q_2 = \frac{(m-r)}{n}$,

$f(z_i) = kz_i^{k-1}\exp(-z_i^k)$, $q_1 = \frac{r}{n}$, $q_2 = \frac{(n-m)}{n}$, $\Gamma(a;b)$ is the incomplete gamma function, Γ' and Γ'' are the first and second derivatives of the gamma function respectively. Holding q_1 and q_2 fixed and letting $n\to\infty$, z_{r+1} converges in probability to $\hat{z}_{r+1}$ where $F(\hat{z}_{r+1}) = \int_0^{\hat{z}_{r+1}} f(t)dt = q_1$, and z_m converges in probability to $\hat{z}_m$ where $1-F(\hat{z}_m) = \int_{z_m}^{\infty} f(t)dt = q_2$ (Ref 12:12-13; 10:558-559).

It should be noted that n , θ , and θ^2 are used to facilitate finding the limits and to make the equations easier to handle. Since n and θ are constants, the

constant factors multiplied by the values of k_{ij} in the information matrix may be temporarily removed in order to make inverting the information matrix easier. The constant factors can be inverted and multiplied by the corresponding elements of the inverted information matrix in order to produce the variance-covariance matrix V. Thus, letting

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} -1 \qquad (20)$$

the elements of V are (Ref 12:4):

$$V = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{a_{11}\theta^2}{n} & \frac{a_{12}\theta}{n} \\ \frac{a_{21}\theta}{n} & \frac{a_{22}}{n} \end{bmatrix} \qquad (21)$$

Lannon's computer program finds the values of the variance-covariance matrix by computing the values of k_{ij}, inverting them, and then multiplying the inverted matrix elements by the appropriate constant factors. This program

is used for both complete and censored samples since a complete sample is just a censored sample with zero samples censored above ($n-m=0$ and thus $q_2=0$) and below ($r=0$ and thus $q_1=0$) .

After Steps 1, 2, and 3 are done for each system component, the bivariate method continues with Step 4 by generating a sample of the component shape parameter and a sample of the component scale parameter. In order to do this, the maximum likelihood estimates of θ and k are used as the means of the bivariate normal distribution of the shape and scale parameter estimates. Using the fact that the statistic $Z = \frac{Y-\mu}{\sigma}$ has a standard normal distribution if Y is an independent normally distributed random variable with mean μ and standard deviation σ , $Y = \sigma Z + \mu$ where Y represents a random sample of each of the two parameter estimates. Thus, the random samples can be found using (Ref 12:10-12):

$$Y = V^{\frac{1}{2}}Z + M \tag{22}$$

where

Y = 2x1 vector containing the two random samples of the shape and scale parameter estimates

V = 2x2 asymptotic variance-covariance matrix which is symmetric

Z = 2x1 vector containing two pseudo-random numbers from the standard normal distribution

m = 2x1 vector containing the two maximum likelihood estimates of the shape and scale parameters

Once the sample parameter estimates are obtained, the bivariate procedure then continues with Steps 5 through 8 as already described.

Lannon analyzed two systems over a mission run time of 75 hours. One system consisted of one component in series with two parallel components, and the other system was a more complex eight-component network. The size of sample data varied among components; and, some component data were censored and some were not. Lannon found the lower confidence limit on the system reliability for four different sizes of simulated system reliability point estimates (99, 499, 999, and 2999) at eleven various confidence levels (99%, 98%, 97%, 96%, 95%, 90%, 85%, 80%, 70%, 60%, and 50%). Because each simulation run was a random sample of system reliability points for only one Monte Carlo run, Lannon pointed out that no conclusion could be drawn as to trends of these lower confidence limits as the number of points increased. He could only state that the lower confidence limit for a 2999-point simulation had to be mathematically more accurate than that for a 99-point simulation (Ref 12: 22-25).

It should be noted that all component reliability estimates belong to the half open interval [0,1) : no component reliability estimate of value one will ever be generated. An examination of Eq (4), which is used to generate all component reliability estimates, indicates

that the only way a component reliability estimate could have a value of one is for (t-c) to equal zero or for $\hat{\theta}$ to approach infinity. However, the quantity (t-c) has a constant value of 100 since t has a value of 100 and c has a value of 0. Also, the parameter θ will never have an estimated value that approaches infinity due to the conditions imposed in the parameter estimation routine PARES. Thus, because no component reliability estimates will ever equal one in value, no system reliability estimate will ever equal one in value. Therefore, Lannon simulated 99, 499, 999, and 2999 system reliability point estimates instead of 100, 500, 1000, and 3000 estimates respectively because the upper reliability bound of one will, with the highest ordered estimated reliability point, form the 100th, 500th, 1000th, or 3000th interval of the reliability range when finding the lower reliability limit in Step 8 of the bivariate technique.

Lannon alludes to the fact that he conducted a brief check of the accuracy of the confidence level (Ref 12:25) by finding confidence interval coverage of the true system reliability as outlined in Steps 8 through 10 of the general Monte Carlo technique. He did this for a Monte Carlo size of 100 runs on both systems, generating 99 system reliability point estimates for each run. He found the confidence level to be reasonably accurate for this least accurate case of 100 Monte Carlo runs and this smallest number of

simulation points (Ref 12:26). He continued his accuracy check by performing 300 and then 700 Monte Carlo runs on his simple three-component system for the 99-point case. He found that, in general, each calculated confidence interval was wider than required, i.e., it covered the true system reliability more often than required by the given confidence level. If the calculated coverage was below the required coverage, the calculated coverage approached the required coverage and surpassed it when more runs were made. If the calculated coverage was above the required coverage, then the calculated coverage approached the required coverage as more runs were made (Ref 12:28-29). Lannon concluded that as the size of component failure data increases and as the number of simulated reliability points increases so does the accuracy of the calculated confidence interval (Ref 12:30).

Univariate Technique

In 1979, Randall Putz developed a univariate Monte Carlo technique to estimate the lower confidence limit of a system's reliability when the component failure times have a Weibull distribution with location parameter c known or equal to zero. Putz's technique is a univariate or one-variable technique even though the other two parameters of the Weibull distribution, θ and k, are unknown and must be estimated for each system component. The single variable used in this technique is $R(t)$, the true component

reliability. The distribution of the component reliability estimate, $\hat{R}(t)$, depends only on the one variable $R(t)$. Thus $\hat{R}(t)$ does not depend directly on the failure times, t, or the maximum likelihood estimates of the unknown parameters, $\hat{\theta}$ and $\hat{k}$, i.e., it depends on them only indirectly through the expression for $R(t)$. Putz used this fact in Step 3 of the general Monte Carlo technique when generating a sample point estimate of the component's reliability.

This fact was discovered by Darrell Thoman, Lee Bain, and Charles Antle in their studies of the Weibull probability distribution and the maximum likelihood estimates of the Weibull parameters. In a paper published in 1970, they showed that for a given component sample size n, the distribution of $\hat{R}(t)$ depends only upon the component's true reliability $R(t)$ (Ref 29:364). This allowed them to study the distribution of $\hat{R}(t)$ empirically and to generate a table of biases for $\hat{R}(t)$ from the empirical distribution. The bias value is tabled by component sample size n and by the true component reliability $R(t)$. They found the biases to be very small (most biases were in the range 0.000 to 0.006; only for $8 \leq n \leq 15$ was the bias larger, the largest value being 0.015) and not worth trying to eliminate (Ref 29: 365). Thus, $\hat{R}(t)$ is close to being an unbiased estimator of $R(t)$.

This three-man team also generated a table of variances for $\hat{R}(t)$ and showed that $\hat{R}(t)$ has a variance that is almost equal to the Cramer-Rao Lower Bound (CRLB), i.e., the lower bound of the variance of the unbiased estimator of $R(t)$ (Ref 29:365-366). In order to compare variances of $\hat{R}(t)$ with the CRLBs for $R(t)$, they generated values for the CRLB by first showing that the CRLB is also a function of $R(t)$ and n. They used the fact that the asymptotic variance of $\hat{R}(t)$ is the CRLB (since the bias of $\hat{R}(t)$ goes to zero as n increases, $\hat{R}(t)$ becomes an unbiased estimator of $R(t)$ as n increases). The equation for the CRLB is

$$CRLB = \sigma^2_{\hat{\theta}}[\frac{\partial R}{\partial \theta}]^2 + 2\sigma_{\hat{\theta},\hat{k}} \frac{\partial R}{\partial \theta} \frac{\partial R}{\partial k} + \sigma^2_{\hat{k}}[\frac{\partial R}{\partial k}]^2 \qquad (23)$$

where $\sigma^2_{\hat{\theta}}$, $\sigma_{\hat{\theta},\hat{k}}$, and $\sigma^2_{\hat{k}}$ are elements of the asymptotic covariance matrix of $(\hat{\theta},\hat{k})$. Then they substituted into Eq (23) the asymptotic covariance matrix element values which they found in a previous study of theirs (Ref 28:449) to be $\sigma^2_{\hat{\theta}} = 1.109\theta^2/(nk^2)$, $\sigma^2_{\hat{k}} = 0.608k^2/n$, and $\sigma_{\hat{\theta},\hat{k}} = 0.257\theta/n$. This resulted in the CRLB being a function of $R(t)$ and n, i.e.,

$$CRLB = R^2(\ln R)^2 \{1.109 - 0.514 \ln(-\ln R) + .608(\ln(-\ln R))^2\}/n \qquad (24)$$

After tabulating values of the CRLBs of $R(t)$ and variances of $\hat{R}(t)$ for 27 cases, Thoman, Bain, and Antle found that the variance of $\hat{R}(t)$ was always greater than the CRLB for low component sample sizes and low $R(t)$ values but equalled the CRLB as n increased and as $R(t)$ increased (Ref 29: 366). They concluded that for large n, one can assume that $\hat{R}(t)$ is normally distributed with mean $R(t)$ and variance equal to the CRLB in order to obtain confidence limits for $R(t)$ (Ref 29:366). Therefore, by showing that $\hat{R}(t)$ is approximately a minimum variance unbiased estimator of $R(t)$, Thoman, Bain, and Antle showed that $\hat{R}(t)$ is a very good estimator of $R(t)$ (Ref 29 363-366).

Although Thoman, Bain, and Antle felt that the bias of $\hat{R}(t)$ was not worth removing, Putz did remove it in his univariate technique in an attempt to be as accurate as possible. He used a table of biases for $\hat{R}(t)$ generated by David Antoon (Ref 1:44) because Antoon's tabled values had a larger number of significant digits than those of the three-man team (Ref 24:15,37). Putz also used Antoon's tabled standard deviation values of $\hat{R}(t)$ for the same reason. Antoon generated the biases and the variances (and thus the standard deviations since standard deviation equals the square root of the variance) via a Monte Carlo simulation procedure similar to that used by Thoman, Bain, and Antle. The standard equation bias = $E(\hat{R}(t))-R(t)$ was used to find the biases, and the equation used to find the variance was

$$\text{Var}[\hat{R}(t)] = \sum_{i=1}^{M} \frac{[E(\hat{R}(t))-R(t)]^2}{M-1}$$

where M is the number of Monte Carlo runs and thus the number of sample points in the $\hat{R}(t)$ distribution (Ref 1: 143-144). Because Antoon used 2000 sample points (Ref 1:143) and Thoman, Bain, and Antle imply that they used 10,000 points (Ref 29:366) to generate the tabled values, Putz compared the tabled values of the two procedures. He found them to be very close in value and attributed the difference in values to the variability in the Monte Carlo procedure itself, i.e., the use of different pseudo random numbers, rather than to the difference in the number of Monte Carlo runs. The table of biases and the table of standard deviations from Antoon's research are presented in Tables I and II respectively.

Putz's computerized univariate technique uses the findings of Thoman, Bain, and Antle regarding $\hat{R}(t)$ and thus expands upon the generalized Monte Carlo technique. The univariate technique consists of ten steps (Ref 24:26-27, 47-65) as follows:

1. Generate a sample of component failure times using the true Weibull distribution parameters θ , k , and c . This is done by calling the International Mathematical & Statistical Libraries (IMSL) routine GGWIB to generate random Weibull deviates from the Weibull shape parameter and

TABLE I

Bias of $\hat{R}(t)$ (Ref 1:44)

R(t)	Sample Size 8	9	10	11	12	13	14	15
.500	.00453	.00480	.00494	.00561	.00577	.00492	.00397	.00405
.550	.01067	.00952	.00944	.01081	.00976	.00767	.00688	.00610
.600	.00842	.00740	.00749	.00763	.00692	.00705	.00639	.00588
.650	.01508	.01267	.01232	.01162	.01088	.00899	.00836	.00806
.700	.01006	.01115	.01034	.00893	.00581	.00645	.00585	.00585
.750	.01064	.00926	.00881	.00816	.00821	.00781	.00728	.00635
.800	.01249	.01245	.01156	.01129	.01092	.01014	.00942	.00941
.850	.00670	.00632	.00530	.00556	.00493	.00597	.00583	.00483
.900	.00388	.00311	.00329	.00326	.00257	.00260	.00149	.00103
.925	.00233	.00384	.00304	.00268	.00200	.00187	.00132	.00208
.950	.00073	.00091	.00062	.00030	.00039	.00025	.00007	-.00026
.980	-.00246	-.00208	-.00232	-.00209	-.00192	-.00158	-.00139	-.00138

TABLE I (continued)

R(t)	Sample Size 20	25	30	40	50	75	100
.500	.00317	.00197	.00097	.00100	.00109	.00071	.00017
.550	.00519	.00388	.00307	.00367	.00310	.00275	.00203
.600	.00345	.00206	.00283	.00292	.00251	.00131	.00021
.650	.00632	.00499	.00369	.00454	.00323	.00261	.00210
.700	.00506	.00432	.00373	.00308	.00204	.00101	.00107
.750	.00459	.00386	.00432	.00343	.00213	.00184	.00054
.800	.00728	.00600	.00538	.00403	.00333	.00148	.00036
.850	.00453	.00262	.00223	.00223	.00114	.00158	.00149
.900	.00084	.00095	.00125	.00165	.00116	.00105	.00079
.925	.00103	.00089	.00004	.00044	-.00004	-.00035	-.00012
.950	.00075	.00070	.00018	.00010	.00015	.00004	.00028
.980	-.00118	-.00088	-.00075	-.00047	-.00029	-.00036	-.00027

TABLE II

Sigma of $\hat{R}(t)$ (Ref 1:42)

R(t)	Sample Size 8	9	10	11	12	13	14	15
.500	.15599	.14746	.13784	.13182	.12539	.11956	.11362	.11131
.550	.15564	.14560	.13740	.13053	.12566	.11704	.11412	.10823
.600	.15009	.14425	.13423	.12764	.12134	.11578	.11296	.10926
.650	.14355	.13650	.12831	.12107	.11685	.11129	.10853	.10439
.700	.13916	.13010	.12165	.11407	.11117	.10664	.10371	.09883
.750	.12562	.11740	.11085	.10684	.10116	.09698	.09278	.09098
.800	.11047	.10436	.09804	.09387	.08761	.08681	.08267	.07919
.850	.09706	.09130	.08776	.08428	.08015	.07652	.07330	.07279
.900	.07634	.07109	.06836	.06476	.06311	.05967	.05803	.05751
.925	.06275	.05706	.05534	.05106	.05116	.04801	.04752	.04534
.950	.04825	.04618	.04334	.04139	.03968	.03796	.03674	.03636
.980	.02776	.02541	.02479	.02288	.02221	.02072	.01922	.01908

TABLE II (continued)

R(t)	Sample Size 20	25	30	40	50	75	100
.500	.09647	.08350	.07636	.06628	.05915	.04735	.04062
.550	.09365	.08227	.07555	.06329	.05738	.04731	.04091
.600	.09356	.08374	.07639	.06312	.05673	.04793	.04108
.650	.09024	.08017	.07250	.06253	.05547	.04514	.04019
.700	.08536	.07462	.06775	.05918	.05494	.04261	.03820
.750	.07947	.07015	.06452	.05548	.04919	.04015	.03539
.800	.06981	.06258	.05661	.04825	.04397	.03704	.03162
.850	.06297	.05557	.05099	.04345	.03901	.03182	.02725
.900	.04739	.04376	.03973	.03397	.03054	.02515	.02214
.925	.04041	.03629	.03265	.02844	.02538	.02125	.01845
.950	.03108	.02789	.02588	.02212	.01995	.01623	.01347
.980	.01613	.01404	.01294	.01109	.00986	.00807	.00701

then by substituting them into the equation $t = \theta$ * deviate + c which is in essence Eq (13).

2. Using the component failure times, find the maximum likelihood estimates of the shape and scale parameters of the component distribution. The IMSL routine ZSYSTM is called to solve the two nonlinear simultaneous maximum likelihood equations for θ and k. ZSYSTM does this by using a Newton-type quadratically convergent method and matrix algebra.

3. Estimate the component reliability by substituting the true location parameter and the maximum likelihood estimates of the shape and scale parameters into the component reliability equation, i.e., Eq (4).

4. Calculate the unbiased component reliability estimate by subtracting the bias from the maximum likelihood estimator of the reliability. The bias is found by interpolating with a cubic spline in Antoon's tabled values using the IMSL routines IBCICU and IBCEVU.

5. Form a vector of component reliability estimates distributed normally about the unbiased estimator by using the following equation which parallels Eq (22):

$$Y = \mu + \sigma Z \qquad (25)$$

where

Y = the desired estimate

μ = the mean or unbiased estimate

σ = the standard deviation of the unbiased estimate

Z = a standard normal deviate

Find the standard deviation of the unbiased estimator using the table of standard deviations from Antoon's research by again using the cubic spline routines. Compare the standard deviation to the square root of the Cramer-Rao Lower Bound and use the larger of the two values as the standard deviation. After the vector of estimates is formed, find the estimates that exceed one in value and set them equal to one since reliability values have an upper bound of one.

6. Repeat Steps 1 through 5 for each component.

7. Calculate a vector of system reliability estimates using the estimates of component reliability and the system reliability equation.

8. Calculate the true reliability of the system using the true component reliabilities (calculated from the true component parameters). Order the vector of system reliability estimates and find the $100(1-\alpha)$ percent lower confidence limits and associated confidence intervals. Compare these to the true system reliability and note if the intervals contain the true system reliability.

9. Repeat Steps 1 to 8 until the desired number of Monte Carlo simulations is reached.

10. Find the percentage of runs in which the confidence intervals covered the true system reliability.

Putz ran 600 Monte Carlo simulations on each of four systems using a mission run time of 100 hours. He found the confidence interval coverage of the true system reliability for five different component sample sizes (10, 15, 20, 50, and 100) at seven different confidence levels (99%, 95%, 90%, 80%, 70%, 60%, and 50%). He found that, for each system, as the component sample size increased, the confidence interval coverage improved. He also noted that the confidence interval coverage was low for the smaller sample sizes. This low coverage results in an optimistic system reliability estimate, i.e., the lower confidence limit being used as the system reliability estimate is too high in value for these particular component sample sizes. He also found that the systems with lower true system reliabilities had more accurate confidence interval coverage and thus more accurate lower confidence limits (Ref 24:32).

Knowing that the component reliability estimates are asymptotically normal from the research by Thoman, Bain, and Antle, Putz conducted a Kolmogorov-Smirnov test of normality on the actual distribution of component reliability estimates. He found that the distribution was normal for component sample sizes of 50 or more. However, for component sample sizes of 20 or less, he found that the distribution was not normal with mean equal to the unbiased estimate of R(t) and standard deviation equal to the interpolated value from Antoon's tables. For these component sample sizes, he

found that the distribution of component reliability estimates was skewed to the left. That is, the distribution of component reliability estimates was significantly affected by the upper reliability bound of one but not by the lower reliability bound of zero. He recommended that further research be done to find the distribution of $\hat{R}(t)$ and suggested that a beta distribution be tried (Ref 24:41,43).

Putz concluded that his univariate method worked best for systems that have component reliabilities and system reliabilities less than 0.9. If the reliabilities are greater than 0.9, skewness of the $\hat{R}(t)$ distributions impacts the results. However, the univariate method can be used in this case if large sizes of component test data are available to use with this technique (Ref 24:43).

Because skewness of the $\hat{R}(t)$ distributions was largely due to generating component reliability estimates greater than one in value and then setting those estimates equal to one, a technique for replacing these values with more realistic reliability values should improve the accuracy of the univariate technique. Such a method of replacement is discussed next.

Gatliffe's Technique

In 1976, Thomas Gatliffe analyzed the accuracy of a log-gamma method for computing lower confidence limits for system reliability using only component data. The system components were assumed to fail independently of one another,

and the test environment under which component data was collected was assumed to approximate the system mission environment. Gatliffe's procedure followed that of the generalized Monte Carlo technique. He assumed that the number of component failures was binomially distributed (Ref 9:7-9,17). His specific Monte Carlo technique is not of interest to this thesis. However, the idea behind his method of compensating for perfect system reliability estimates is of interest because of the component reliability estimates--and thus some system reliability estimates--of value one generated by Putz in his univariate technique. In order to correct for the unrealistic situation of perfect system reliability, Gatliffe viewed the system as a series network and replaced one of the simulated component data of zero failures with 0.37 failures if the desired significance level α was 0.2 in value, 0.25 failures if α was 0.1, and 0.16 failures if α was 0.05 (Ref 9:15). These failure values used by Gatliffe are the average solution F_i^* to the following equation (Ref 9:46):

$$(F_i^*)^2 + (2N-3)F_i^* + (N-1) \ln \alpha(\chi^2_{(2,1-\alpha)}) = 0 \qquad (26)$$

where N is the component sample size. The derivation of this equation is presented in Appendix A.

If any system network were divided into subsystems that were all in series with one another, Gatliffe's technique could be used to replace any simulated subsystem

reliability estimate of value one with a more realistic value. Such a procedure may improve the accuracy of the univariate technique. To apply the technique, component reliability estimates still need to be considered as discussed in the next section.

III. Procedure

This section describes modifications and improvements made to the three Monte Carlo techniques of estimating system reliability and the common design parameters used in order to compare the accuracy of the techniques.

Systems Analyzed

The univariate and bivariate system reliability estimation techniques were tested on four simple system networks: one with strictly serial components (System 1), one with only parallel components (System 3), and two with simple combinations of series and parallel components (Systems 2 and 4). These four combinations of the same components were used in order to determine whether the overall accuracy of a technique depends upon a particular network configuration (Ref 24:23). Each system contained three to five components as illustrated in Figure 4. The reliability of a more complex system network can be found by dividing the network into subsystems similar to these and by applying Bayes rule.

For all four systems, each component was assumed to have failure times that followed a Weibull distribution, i.e., the failure times can be modeled by Eq (1). Parameter values for the five components are:

1. Component 1: $k=2$, $\theta=250$, $c=0$
2. Component 2: $k=3$, $\theta=210$, $c=0$

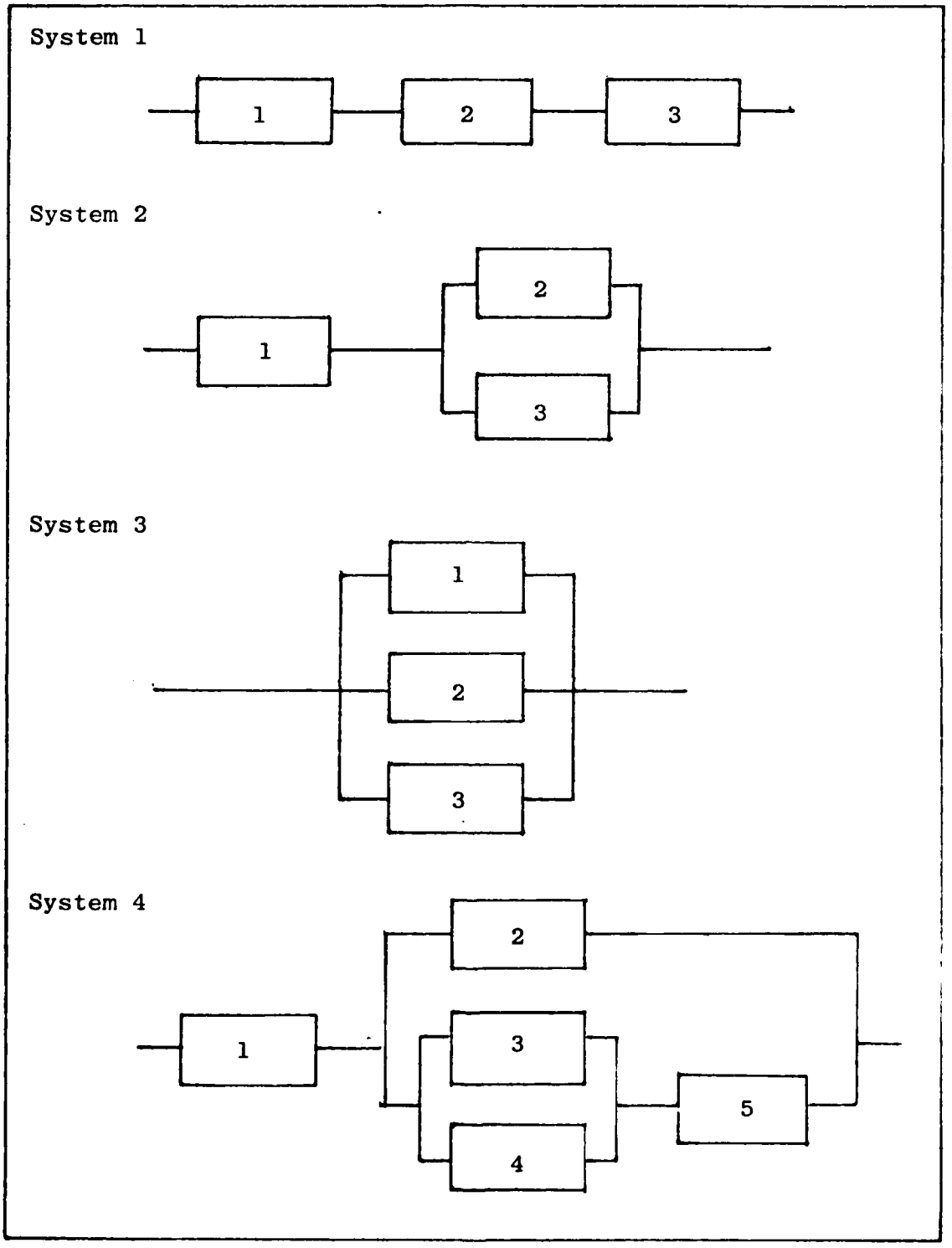


Fig 4. Systems 1, 2, 3, and 4 (Ref 24:24)

3. Component 3: k=2, θ=300, c=0

4. Component 4: k=3.5, θ=150, c=0

5. Component 5: k=2.5, θ=250, c=0

Mission run time for each component is arbitrarily assumed to be 100 hours. These component parameters are the exact parameters used by Putz in his research (Ref 24:22-23).

Because the original intent of this thesis was to use Putz's results to represent the univariate-normal technique, the same component parameters, sample sizes, mission time, confidence levels, and Monte Carlo runs were used. Thus each technique generated test data (component failure times) for the five component sample sizes of 10, 15, 20, 50, and 100 and tested confidence interval coverage for the seven confidence levels of 99%, 95%, 90%, 80%, 70%, 60%, and 50% over 600 Monte Carlo simulation runs. However, part of the univariate-normal technique had to be recoded and thus Putz's results could no longer be used as a representative case of the univariate-normal technique. In addition, the excessive central processing time required to simulate the same number of system reliability points (2000) as Putz used led to the simulation of lower numbers of reliability points in each of the techniques. For the univariate-normal, univariate-beta, and bivariate techniques, 100, 200, 600, and 1000 system reliability point estimates were generated for each component sample size, and confidence interval coverage of the true system reliability was found.

When recoding of the univariate-normal technique was found to be needed, the numbers used for sample sizes, confidence levels, and Monte Carlo runs were not changed since they have good underlying theoretical and empirical bases. As already stated in Section II, researchers have found that 20 to 100 components should be tested in order to get the most information from a testing situation. Also, interest exists in reducing these numbers of components tested in order to reduce costs. Therefore, the sample sizes of 10, 15, 20, 50, and 100 are appropriate for this research project. It was expected that the higher component sample sizes would lead to better confidence interval coverage of the true system reliability because better estimates should result from having more information, i.e., more failure times on which to base the estimates.

Confidence levels above 50%, especially in the 90% range, are appropriate since a confidence level indicates the degree of certainty associated with an estimate. The military services obviously would like to have high degrees of certainty, and thus low degrees of risk, attached to system reliability estimates.

The use of 600 Monte Carlo simulation runs is also appropriate and should produce an estimation error of 4%. This error is measured by the variance of the estimates due to the Monte Carlo simulation process; it does not include the variance due to any other design parameters in the

technique. Assuming that the random numbers generated are truly random and the system model is perfect, the only errors in a Monte Carlo simulation are due to sampling. These sampling errors decrease with $1/\sqrt{n}$ where n is the number of trials or runs of the simulation (Ref 27:258). Thus, if n is 600, $1/\sqrt{n}=.0408$ or 4.08% error. For n=1000, $1/\sqrt{n}=.0316$ or 3.16% error, and for n=10,000, $1/\sqrt{n}=.01$ or 1% error. The additional processing time needed for 1,000 runs was not judged to be worth the extra 0.92% accuracy, and the time needed for 10,000 runs was certainly not worth the extra 3% accuracy.

System parameters are further discussed under the procedures for each particular technique.

Determination of the Accuracy of Each Technique

The purpose of each technique is to find the lower confidence limit of system reliability and to use this limit as an estimate of the true reliability of the system. (The lower confidence limit of system reliability will be referred to as "the system reliability estimate" or "the estimate" in the following pages.) In order to judge the accuracy of the system reliability estimates produced by each technique, the true system reliability was calculated and compared to each estimate as outlined in Steps 8 through 10 of the general Monte Carlo procedure discussed in Section II.

The true system reliability was found by first calculating the true reliabilities of each component in the

system network and then using the appropriate reliability expression for series and parallel component networks given in Figure 1. The true reliability of each component was found by substituting its known parameters, given in this section, into the reliability equation for a component whose failure times have a Weibull distribution (Eq (4)). The resulting true component reliabilities, R_j for j=component 1 to component 5 , are:

$$R_1 = \exp(-(100/250)^2) = .85214$$

$$R_2 = \exp(-(100/210)^3) = .89765$$

$$R_3 = \exp(-(100/300)^2) = .89484$$

$$R_4 = \exp(-(100/150)^{3.5}) = .78512$$

$$R_5 = \exp(-(100/250)^{2.5}) = .90376$$

Then the reliability of each system, Rs_i where i=system 1 to system 4 , was calculated as follows:

$$\begin{aligned} Rs_1 &= R_1R_2R_3 \\ &= (.85214)(.89765)(.89484) \\ &= .68448 \end{aligned} \tag{27}$$

$$\begin{aligned} Rs_2 &= R_1(1-\bar{R}_2\bar{R}_3) \\ &= (.85214)(1-(1-.89765)(1-.89484)) \\ &= .84297 \end{aligned} \tag{28}$$

$$\begin{aligned} Rs_3 &= 1-\bar{R}_1\bar{R}_2\bar{R}_3 \\ &= 1-(1-.85214)(1-.89765)(1-.89484) \\ &= .99841 \end{aligned} \tag{29}$$

$$Rs_4 = R_1(1-\bar{R}_2(1-R_5(1-\bar{R}_3\bar{R}_4)))$$

$$= (.85214)(1-(1-.89765)(1-.90376(1-(1-.89484)(1-.78512)))$$
$$= .84197 \qquad (30)$$

During each Monte Carlo run, a counter kept track of the number of times a confidence interval covered the true system reliability. The resulting percent coverage is an indication of the confidence level that can be attached to that system reliability estimate. Because a confidence interval was constructed for a given confidence level (i.e., for a confidence level of 99%, 95%, 90%, 80%, 70%, 60%, or 50%), the calculated percent confidence interval coverage should closely approximate the given confidence level if the estimation technique is accurate.

If the confidence interval covered the true system reliability a smaller percentage of times than would be indicated by a given confidence level, the confidence interval coverage is said to be optimistic. This means that the system reliability estimate is higher than the true system reliability more often than desired (more often than $100(\alpha)\%$ of the times). Therefore, the given confidence level cannot be associated with a system reliability estimate generated by the particular technique. How much the confidence interval coverage deviates from the given confidence level will determine the accuracy of the technique in estimating system reliability.

If the confidence interval covered the true system reliability a larger percentage of times than would be

expected from a given confidence level, then the confidence interval coverage is said to be conservative. This means that the system reliability estimate is lower than the true system reliability more often than the given confidence level would indicate and that a confidence level at least as high as the given confidence level can be attached to the reliability estimate. Thus, conservative confidence interval coverage is a very desirable result. Again, the amount the confidence interval coverage deviates from the given confidence level determines the accuracy of the technique.

Bivariate Method of Generating System Reliabilities

To test the accuracy of the bivariate technique of estimating system reliability, only minor modifications were made to Lannon's computer programs. Lannon's programs were designed to find the lower confidence limit of system reliability for a given confidence level. Computer code was added to repeat this process 600 times and to determine the confidence interval coverage of the true system reliability for each given confidence level. Modifications were also made to take care of parameter estimates and reliability estimates exceeding the ranges of exponential and natural logarithm functions. In addition, the seed for the generation of uniform and normal random numbers was set to the computer clock time in order to make each simulation run independent from the other runs.

A listing of the computer code for the modified bivariate technique, along with definitions of the major variables, is provided in Appendix B.

Univariate Method of Generating System Reliabilities

As stated at the beginning of this section, a portion of Putz's univariate-normal technique had to be recoded making the use of his results in the comparison of techniques invalid. The portion replaced was the method of finding the maximum likelihood estimates of the shape and scale parameters of the Weibull distribution. As described in Step 2 of the univariate technique, Putz used the IMSL routine ZSYSTM to solve the two nonlinear maximum likelihood equations for θ and k. However, this IMSL routine was unable to solve the equations due to its constant inability to invert a matrix it formed in the process. As a result, this approach was replaced with the iterative parameter estimation routine developed by Harter and Moore and used by Lannon in his bivariate technique.

Other minor modifications were made to the code such as deletion of dead code, deletion of code extraneous to the technique (i.e., code to perform a Kolmogorov-Smirnov test), and the addition of code to take care of reliability estimates exceeding the ranges of the exponential function. Also, two seeds used in random number generators were varied rather than using the two constant seed values in Putz's original code. Although Putz states that he checked to see

if his results were sensitive to the choice of an initial random number seed and found that they were not, he implies that he only checked the seed on the generator of random normal deviates and not on the generator of failure times (Ref 24:36-37). Thus, the seeds were changed for the same reason as stated for the bivariate case--to make each simulation run independent from the other runs.

The seeds were not set to the clock time as was done in the bivariate case. There were two reasons for this: (1) two seeds existed and should not be the same clock value even though they were used in different random number generators, and (2) the comparison of techniques includes a direct comparison of the univariate-normal and univariate-beta techniques; thus, the same seeds needed to be used in the two techniques so that only the underlying distribution of the component reliability estimates varied. Random values of seeds were selected from a table of random numbers (Ref 26:370). A different seed value was used for each component sample size and for each set of Monte Carlo runs when all 600 runs could not be made at once. The seeds used in all cases are listed in Appendix C.

The final minor modification made to Putz's original code is the setting of negative component reliability estimates to zero since negative values do not exist in reality. This modification was needed because Eq (25), used in Step 5 of the univariate-normal technique, not only produces

component reliability estimates with values greater than one but also produces estimates with values less than zero. A counter was also added to keep track of the number of times negative estimates are generated to see if the number is significant enough to impact the results. Thus, the univariate-normal technique used consisted of the 10 steps described in Section II with Steps 2 and 5 replaced as follows:

2. Using the component failure times, find the maximum likelihood estimates of the shape and scale parameters of the component distribution. The subroutine PARES is called to estimate the parameters using an iterative linear interpolation technique and the maximum likelihood equations of the parameters.

5. Form a vector of component reliability estimates distributed normally about the unbiased estimator by using the following equation:

$$Y = \mu + \sigma z$$

where Y is the desired estimate, μ is the mean or unbiased estimate, σ is the standard deviation of the unbiased estimate, and z is a standard normal deviate. The standard deviation of the unbiased estimator is found using the table of standard deviations from Antoon's research by again using the cubic spline routines. However, the standard deviation is compared to the square root of the

Cramer-Rao Lower Bound and the larger of the two values is used as the standard deviation. After the vector of estimates is formed, find the component reliability estimates that fall outside the closed interval [0,1] and set them equal to the lower bound of zero or the upper bound of one, whichever value each is nearest to numerically.

A listing of the computer code for the modified univariate-normal technique is provided in Appendix C.

Beta Distribution of Component Reliabilities

As discussed in Section II, Putz generated sample component reliability point estimates (Step 5 of the univariate-normal technique) by assuming the unbiased maximum likelihood estimate of the component reliability was the mean of a normal distribution. He then conducted a Kolmogorov-Smirnov test for normality on his component reliability estimates and found that, although he assumed that the estimates were asymptotically normal, their distribution was not normal. He also found that the distribution was not symmetric but skewed toward the upper reliability bound of one (Ref 24:41). Because Putz obtained this non-symmetric distribution, a modification to his univariate-normal technique was made. Another model, the beta distribution, for the pattern of component reliability estimates was chosen. This new distribution not only should better model the pattern of component reliability estimates, but

it will also avoid the generation of estimates larger than one or less than zero in value. It is expected that this change will improve the accuracy of the univariate technique.

The beta distribution was chosen to model the pattern of component reliability estimates since its graph can assume a large variety of shapes for various values of its parameters. The probability distribution function for the beta random variable Y is

$$f(y) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}, & \alpha,\beta>0 \\ & 0\leq y\leq 1 \\ 0 & \text{, elsewhere} \end{cases} \qquad (31)$$

with mean $= \alpha/(\alpha+\beta)$ and variance $\text{var} = (\alpha\beta)/[(\alpha+\beta)^2(\alpha+\beta+1)]$ (Ref 19:130,A24). As in the univariate-normal technique, the unbiased maximum likelihood estimate of component reliability was used as the mean of the distribution and the corresponding interpolated value from Antoon's tables was used as the standard deviation of the distribution. Using these two values, the parameters of the beta distribution, α and β, were found as follows:

$$\text{mean} = \alpha/(\alpha+\beta)$$

$$\alpha\cdot\text{mean} + \beta\cdot\text{mean} = \alpha$$

$$\alpha = \beta(\text{mean}/(1-\text{mean}))$$

$$\alpha = \beta\cdot Q \quad \text{where } Q = \text{mean}/(1-\text{mean})$$

$$\text{Var} = (\alpha\beta)/[(\alpha+\beta)^2(\alpha+\beta+1)]$$

$$\text{Var}(\alpha+\beta)^2(\alpha+\beta+1) = \alpha\beta$$

$$\text{Var}(\beta Q+\beta)^2(\beta Q+\beta+1) = \beta Q\beta \quad \text{where } \alpha = \beta Q$$

$$\text{Var}\ \beta^2(Q+1)^2(\beta(Q+1)+1) = \beta^2 Q$$

$$\text{Var}(Q+1)^2(\beta(Q+1)+1) = Q$$

$$\text{Var} = Q/[(1+Q)^2(\beta(Q+1)+1)]$$

$$\beta(Q+1) = Q/((1+Q)^2\text{Var}) - 1$$

$$\beta = Q/((1+Q)^3\text{Var}) - 1/(Q+1) \qquad (32)$$

$$\alpha = \beta Q$$

$$\alpha = Q^2/[(1+Q)^3\text{Var}] - Q/(Q+1) \qquad (33)$$

The IMSL routine GGBTR was given these values of α and β as input. For each α and β input, the routine output a random variable belonging to the closed interval $[0,1]$. This random beta deviate was set equal to the random sample reliability estimate. Thus, all component reliability estimates were generated by assuming they had a beta distribution with mean equal to the maximum likelihood reliability estimate and standard deviation sigma.

This procedure, now known as the univariate-beta technique, therefore only altered one step, Step 5, of the univariate-normal technique. Thus, the univariate-beta technique consists of the 10 steps for the univariate-normal technique as described in this section with Step 5 replaced as follows:

5. Form a vector of component reliability estimates that have a beta distribution with mean equal to the unbiased estimator and standard deviation equal to that of the unbiased estimator. The standard deviation of the unbiased estimator is found using the table of standard deviations from Antoon's research by again using the cubic spline routines. However, the standard deviation is compared to the square root of the Cramer-Rao Lower Bound and the larger of the two values is used as the standard deviation.

Thus, the modified code for the univariate-normal technique was used for this process with only one change: the code to generate sample estimates of the component reliability. A listing of the code for the univariate-beta technique is provided *in Appendix C.*

Application of Gatliffe's Method

Because many component reliability estimates have values of one due to truncation in the univariate-normal technique, the possibility of having several system reliability estimates with values of one exists. Because perfect system reliability is unrealistic, Gatliffe's technique of adjusting for perfect system reliability estimates was applied to the univariate-normal technique. It was applied to the Monte Carlo simulation of 600 system reliability points for all component sample sizes and all confidence levels. Gatliffe's technique was not applied to the univariate-beta technique because the probability of a

component's reliability being equal to one is zero in the beta case. Although a possibility of some component reliability estimates having values of one exists due to computer roundoff in generating beta deviates, the number of times this would happen is close to zero. Thus, little or no improvement would be made to the accuracy of the univariate-beta technique by applying Gatliffe's method. Gatliffe's method also did not need to be applied to the bivariate case because all component reliability estimates generated belonged to the interval $[0,1)$, as discussed in Section II. Since no component reliability estimates could have a value of one, it is impossible for the system reliability to ever equal one in value.

To apply Gatliffe's technique, each of the four systems was divided into subsystems connected in series. Each subsystem consisted of one component or a grouping of components. Thus, System 1 consisted of 3 subsystems connected in series; subsystems 1, 2, and 3 corresponded to components 1, 2, and 3 respectively. System 2 was divided into 2 serial subsystems; subsystem 1 corresponded to component 1 and subsystem 2 corresponded to components 2 and 3 in parallel. System 3, having three parallel components, consisted of one subsystem in series with itself. Similarly, System 4 consisted of 2 subsystems connected in series; subsystem 1 corresponded to component 1 and subsystem 2 corresponded to

the group of components 2 through 5. These divisions of the four systems into serially connected subsystems are illustrated in Figure 5.

To determine if a system reliability estimate had a value of one and needed to be corrected, the entire expression was not computed, flagged, and recomputed. Instead, the system reliability expressions, Eqs (27) through (30), were analyzed to find which components needed to have a value of one in order to make the system reliability value also equal to one. Using the reliability expressions discussed in Section II and listed in Figure 1, it was noted that in order to have perfect system reliability, each subsystem must have perfect reliability since the reliability expression for a series of "components" is simply the product of the reliabilities of all "components." Therefore, in order for System 1 to have a reliability value of one, all three components must have a reliability value of one. For System 2 to have perfect reliability, subsystems 1 and 2 must have perfect reliabilities. This means that component 1 must have a reliability value of 1 and either component 2 or component 3 must have an estimated reliability value of 1. Because System 3 has one subsystem made up of 3 parallel components, it will have a perfect system reliability value if one or more of the three components has a reliability value of one. Similarly, System 4 will achieve a reliability value of one if component 1 has a reliability value of one

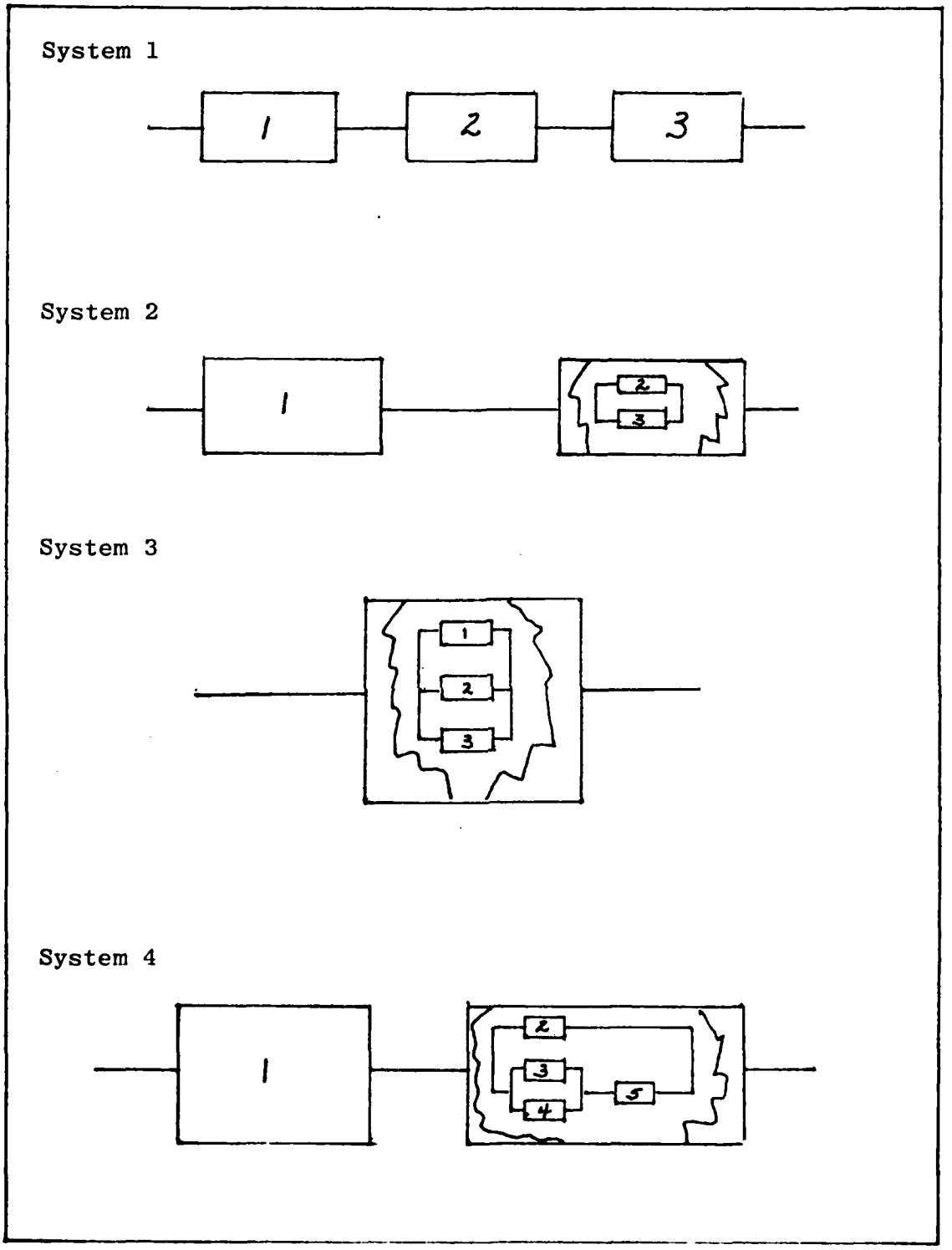


Fig 5. Serial Subsystems of Systems 1, 2, 3, and 4

and either component 2 alone or component 5 plus either component 3 or 4 have perfect reliability values. These relationships can be found by setting each system reliability equation, i.e., Eqs (27) through (30), to one and solving for all possible component values.

If these key components did have estimated values of one, the corresponding system reliability expression was not used to find the system reliability value since it would equal one. Instead, the system reliability was immediately equated to an adjusted value. It should be noted that System 3 has the best chance for increased confidence interval coverage of the true system reliability since it has the best chance for reliability estimates to be adjusted. Only one component needs to have an estimated reliability value of one to produce a reliability value of one for the entire system. Then, Systems 2, 4, and 1--in that order--are likely to need their reliability estimates adjusted. However, the chances of the latter happening are small. Thus, Gatliffe's technique is expected to significantly improve the accuracy of the univariate-normal technique only for System 3.

In order to adjust the system reliability estimates in the case of perfect reliability values of one, the three step procedure suggested by Gatliffe was used. It was assumed that the natural logarithm of the system reliability had a gamma distribution and that the number of failures of each subsystem is distributed binomially. The steps and

their application to the univariate-normal technique are:

1. Find the series component or subassembly with the smallest actual or equivalent number of trials, N or N' (Ref 9:15-16). For each Monte Carlo simulation performed with the univariate technique, all components had the same sample size. It is assumed that the subsystem sample size is therefore equal to the sample size of any one component in the subsystem. Since this results in all serially connected subsystems having the same sample size, any subsystem could be selected as having the smallest sample size.

2. Change the number of component or subassembly failures to F' by using the average values of .37 failures for F' if the significanc level α=.2 , .1 failures for F' if α=.1 , and .16 failures for F' if α=.05 (Ref 9: 15-16). In order to be more exact in replacing failure data values, the value of F^* in Eq (26) was calculated for all possible combinations of α and N that were used in the univariate technique. Thus the replacement value of the number of failures was selected from Table III based on the particular component sample size N and the $1-\alpha$ confidence level being investigated. The values in Table III were computed by using the general solution to Eq (26), i.e.,

$$F^* = \frac{-2N+3 \pm \sqrt{(2N-3)^2-4(N-1)\ln\alpha(\chi^2_{2,1-\alpha})}}{2} \qquad (34)$$

and the component sample size N , the significance level

TABLE III

Adjusted Failure Data

Significance Level	Sample Size 10	15	20	50	100
.01	.0488	.0478	.0474	.0467	.0464
.05	.1610	.1584	.1570	.1550	.1542
.10	.2531	.2493	.2475	.2445	.2435
.20	.3721	.3675	.3652	.3615	.3603
.30	.4431	.4382	.4359	.4319	.4307
.40	.4820	.4771	.4747	.4707	.4694
.50	.4943	.4893	.4870	.4830	.4817

α , and the $\chi^2_{2,1-\alpha}$ values which were obtained from the IMSL routine MDCHI.

3. Recompute the lower confidence limit estimate using the revised failure data (Ref 9:16). In order to change the failure data of the selected component or subsystem, one would change the estimated component or subsystem reliability value to $1-F^*/N$ per Gatliffe's procedure. However, since the univariate technique was designed to have all component sample sizes equal, $1-F^*/N$ will be the same value regardless of which serially connected subsystem is chosen; and, all the rest of the serially connected subsystems will have values of one. Thus, in the univariate technique, the system reliability itself can be changed from a value of one to a value of $1-F^*/N$ for the appropriate F^* corresponding to a particular N and α .

In summary, the univariate technique with Gatliffe's adjustment for perfect system reliability follows the 10 step modified procedure for the normal case with Steps 7 through 10 modified as follows:

7. Calculate the true reliability of the system using the true component reliabilities which are calculated from the true component parameters.

8. For a given confidence level $1-\alpha$, calculate a vector of system reliability estimates using the estimates of component reliability and the system reliability equation. Replace all system reliability estimates of one with a value of $1-F^*/N$, where N is the common component sample size and

$$F^* = \frac{-2N+3 \pm \sqrt{(2N-3)^2-4(N-1)\ln\alpha(\chi^2_{2,1-\alpha})}}{2}$$

9. Order the vector of system reliability estimates and find the $100(1-\alpha)$ percent lower confidence limit and associated confidence interval. Compare this limit to the true reliability and note if the interval contains the true system reliability.

10. Repeat Steps 8 and 9 until all desired confidence levels have been used.

11. Repeat Steps 1 through 10 until the desired number of Monte Carlo simulations is reached.

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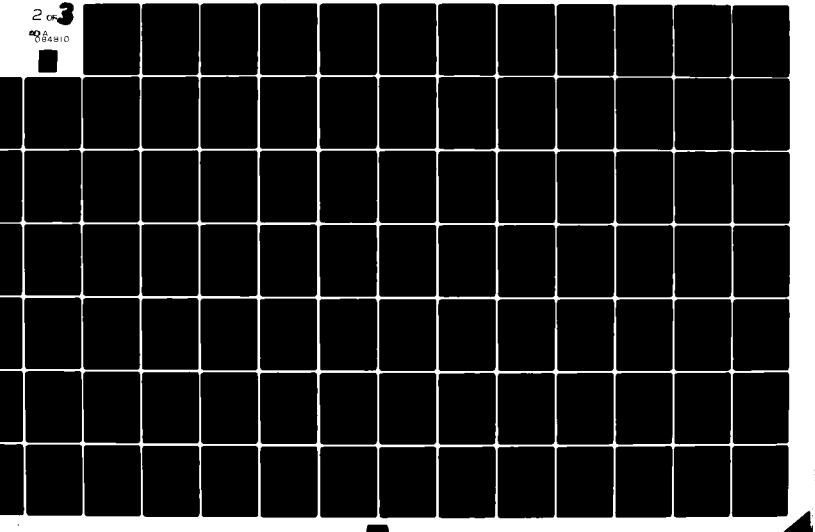

12. Find the percentage of runs in which the confidence intervals covered the true system reliability.

A listing of the computer code for this procedure is provided in Appendix D.

IV. Results and Conclusions

Presentation of Results

As discussed in Section III, a lower confidence interval was constructed for a given confidence level of 99%, 95%, 90%, 80%, 70%, 60%, or 50% (these confidence levels are referred to in the tables and text as the required confidence levels) during each Monte Carlo simulation run. After all Monte Carlo runs were completed, the percentage of times the confidence intervals covered the true system reliability was calculated. This percent coverage should closely approximate the required confidence level if the estimation technique is accurate.

Tables IV through XIX present the percent confidence interval coverages for each of the four systems obtained from the bivariate technique. In general, for the 99, 199, 599, and 999 generated system reliability points, the bivariate technique results in percent confidence interval coverages for System 1 that are higher than the required confidence levels. For Systems 2, 3, and 4, the bivariate technique generally results in coverages being lower than the required confidence levels except for a few cases involving component sample size 100. System 3 has the lowest amount of coverage and Systems 2 and 4 have about the same amount of coverage.

Tables XX through XXV present the percent confidence interval coverage results from using the univariate-normal technique on each of the four system networks. For the

TABLE IV

Percent Confidence Interval Coverage for System 1 Using the Bivariate Technique and 99 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	100.00	99.83	99.83	99.33	99.83
95	98.83	97.50	97.00	94.67	94.67
90	96.17	94.17	93.33	90.00	89.67
80	86.17	85.83	84.17	82.50	82.00
70	76.83	75.67	76.33	72.00	73.17
60	67.50	68.17	65.67	60.33	63.17
50	55.67	58.17	53.17	50.83	52.83

TABLE V

Percent Confidence Interval Coverage for System 2 Using the Bivariate Technique and 99 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.00	98.83	99.83	98.50	99.50
95	93.00	93.50	94.83	94.50	95.83
90	85.33	87.67	89.50	87.50	91.00
80	73.67	75.17	77.83	73.33	80.67
70	63.83	66.00	67.50	63.83	70.33
60	54.33	56.50	55.83	53.50	60.33
50	44.67	47.17	44.33	43.67	50.50

TABLE VI

Percent Confidence Interval Coverage for System 3 Using the Bivariate Technique and 99 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	96.67	96.50	97.50	98.17	98.67
95	83.83	87.83	88.33	90.17	91.17
90	71.67	77.17	81.00	82.50	84.50
80	56.83	63.33	64.50	69.50	74.33
70	46.00	51.33	50.50	56.83	62.33
60	34.83	40.83	39.67	47.00	53.33
50	27.00	32.00	30.50	37.17	43.33

TABLE VII

Percent Confidence Interval Coverage for System 4 Using the Bivariate Technique and 99 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.17	98.83	99.83	98.17	99.67
95	93.17	93.67	95.00	94.83	95.83
90	85.17	88.17	89.17	89.00	91.17
80	74.33	77.17	77.83	74.50	80.83
70	64.50	66.50	68.00	64.17	69.83
60	53.67	57.17	54.83	54.50	60.17
50	43.67	47.17	45.17	44.33	50.83

TABLE VIII

Percent Confidence Interval Coverage for System 1 Using the Bivariate Technique and 199 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	100.00	99.00	99.83	99.67	99.17
95	97.67	96.17	97.50	96.50	97.17
90	95.33	92.50	93.50	91.67	92.50
80	87.00	84.67	84.83	81.67	82.50
70	77.17	74.67	77.67	74.83	72.50
60	68.83	67.67	67.83	64.17	63.50
50	60.17	57.67	58.33	53.67	53.50

TABLE IX

Percent Confidence Interval Coverage for System 2 Using the Bivariate Technique and 199 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	98.83	98.67	99.00	99.00	98.83
95	94.00	93.00	94.83	95.17	95.17
90	88.83	87.33	89.00	90.17	89.50
80	78.33	77.33	77.50	78.00	79.67
70	66.33	68.00	67.83	67.50	70.00
60	55.50	56.50	57.00	58.50	61.17
50	46.00	46.33	46.83	46.17	48.83

TABLE X

Percent Confidence Interval Coverage for System 3 Using the Bivariate Technique and 199 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	95.17	95.50	97.17	98.00	98.17
95	83.50	85.17	88.33	91.50	92.67
90	73.83	77.33	80.50	82.67	85.50
80	59.67	62.17	68.33	70.67	74.00
70	46.33	50.83	56.50	60.00	63.67
60	38.33	41.67	43.83	49.33	51.00
50	30.00	31.67	34.17	39.33	44.00

TABLE XI

Percent Confidence Interval Coverage for System 4 Using the Bivariate Technique and 199 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	98.67	98.67	99.00	99.00	98.33
95	94.33	93.67	94.00	94.83	95.50
90	89.17	87.50	88.00	90.00	90.00
80	78.00	78.00	78.17	78.50	79.67
70	67.00	68.33	67.83	67.17	69.33
60	55.33	57.00	57.00	56.67	59.83
50	44.67	45.83	46.83	46.67	49.00

TABLE XII

Percent Confidence Interval Coverage for System 1 Using the Bivariate Technique and 599 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.83	99.83	99.50	99.00	99.33
95	99.00	97.67	98.17	96.00	96.33
90	96.00	93.67	93.33	92.00	90.83
80	87.00	86.33	85.67	83.17	81.00
70	78.83	78.00	76.17	75.50	69.00
60	69.00	68.50	66.33	66.67	59.67
50	58.33	58.17	54.50	55.33	50.00

TABLE XIII

Percent Confidence Interval Coverage for System 2 Using the Bivariate Technique and 599 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.00	98.83	99.33	99.17	99.17
95	94.50	94.00	95.17	94.50	94.17
90	87.00	88.67	89.83	90.17	88.17
80	74.33	76.50	76.83	78.50	77.83
70	64.67	66.33	67.67	69.50	68.50
60	54.50	57.00	58.33	58.67	58.83
50	46.67	46.83	49.17	48.00	50.00

TABLE XIV

Percent Confidence Interval Coverage for System 3 Using the Bivariate Technique and 599 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	96.83	96.67	98.00	97.33	98.50
95	85.00	87.33	88.00	91.00	93.17
90	74.00	79.67	78.50	84.00	84.33
80	58.00	63.50	64.33	73.17	72.67
70	46.67	52.17	50.67	62.33	58.83
60	36.00	41.00	40.67	49.33	51.17
50	26.83	32.50	33.00	38.67	40.67

TABLE XV

Percent Confidence Interval Coverage for System 4 Using the Bivariate Technique and 599 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.00	99.33	99.17	99.33	99.17
95	94.17	93.67	95.17	94.67	93.83
90	87.17	88.00	89.83	90.00	87.67
80	74.00	76.83	77.50	78.67	78.33
70	65.17	66.83	67.33	68.83	68.17
60	55.67	56.83	59.50	58.67	58.50
50	46.33	47.17	49.00	47.67	49.17

TABLE XVI

Percent Confidence Interval Coverage for System 1 Using the Bivariate Technique and 999 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.83	99.50	99.50	99.50	99.67
95	97.17	97.50	96.50	96.50	97.33
90	93.17	93.50	93.17	92.67	91.33
80	84.50	85.67	84.33	82.67	81.67
70	75.33	76.00	74.67	71.83	72.83
60	67.67	66.50	64.67	60.83	61.50
50	58.17	55.83	57.50	52.50	49.50

TABLE XVII

Percent Confidence Interval Coverage for System 2 Using the Bivariate Technique and 999 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.00	99.33	98.83	98.17	99.17
95	94.00	93.83	93.00	93.33	95.33
90	89.17	87.67	86.33	89.00	91.83
80	77.33	76.33	73.67	79.17	80.17
70	65.00	68.33	64.33	68.33	67.67
60	53.17	59.17	55.50	60.17	56.50
50	45.83	47.17	46.67	49.83	45.83

TABLE XVIII

Percent Confidence Interval Coverage for System 3 Using the Bivariate Technique and 999 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	93.67	96.17	97.00	98.17	99.17
95	81.17	87.17	88.33	91.17	93.83
90	70.50	77.67	78.17	83.33	85.83
80	56.50	63.00	63.33	68.50	73.83
70	46.33	51.17	52.67	58.00	61.50
60	35.17	40.67	39.17	46.67	47.83
50	27.50	32.33	29.67	36.83	37.83

TABLE XIX

Percent Confidence Interval Coverage for System 4 Using the Bivariate Technique and 999 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	99.67	99.33	98.67	98.50	99.00
95	94.33	93.83	93.00	94.00	95.67
90	89.00	87.50	85.83	88.50	91.67
80	76.83	76.67	73.83	79.17	81.17
70	64.33	69.00	64.67	68.67	68.33
60	53.50	58.33	55.50	60.00	58.33
50	45.83	48.33	47.67	48.67	45.50

TABLE XX

Percent Confidence Interval Coverage for System 1 Using the Univariate-Normal Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	95.67	96.17	97.67	97.83	99.17
95	89.50	90.33	93.00	93.17	94.33
90	84.00	85.67	88.67	87.33	88.50
80	75.33	77.17	79.00	77.83	78.83
70	69.17	68.83	67.83	66.83	70.67
60	60.83	59.00	57.00	58.50	61.83
50	51.33	48.67	47.33	46.17	52.17

TABLE XXI

Percent Confidence Interval Coverage for System 2 Using the Univariate-Normal Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	88.67	92.50	92.83	96.00	97.67
95	82.00	87.17	86.33	90.17	93.17
90	78.00	83.00	81.50	85.50	88.50
80	69.33	74.00	73.50	73.83	80.00
70	61.50	64.33	64.67	63.67	71.33
60	52.67	55.33	55.67	55.17	62.00
50	45.33	46.33	48.83	47.00	52.67

TABLE XXII

Percent Confidence Interval Coverage for System 3 Using the Univariate-Normal Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	82.50	87.00	92.67	95.67	97.67
95	69.67	76.33	81.33	87.33	89.83
90	60.67	68.17	71.17	78.17	83.17
80	49.50	52.83	55.83	64.67	70.33
70	38.67	42.83	44.33	52.00	60.00
60	28.50	31.33	35.67	40.17	52.17
50	20.00	23.00	25.67	31.50	42.67

TABLE XXIII

Percent Confidence Interval Coverage for System 4 Using the Univariate-Normal Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	89.00	92.83	92.83	95.67	97.17
95	82.17	87.83	87.17	90.00	93.17
90	77.83	82.33	82.33	85.17	89.17
80	69.33	74.33	73.17	73.67	79.67
70	60.67	65.33	65.00	63.67	71.17
60	53.00	55.17	56.50	54.50	62.00
50	45.17	46.83	49.00	46.67	52.50

TABLE XXIV

Percent Confidence Interval Coverage for System 1 Using the Univariate-Normal Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	94.17	94.67	96.33	97.33	97.33
95	87.83	89.17	90.67	92.67	93.50
90	83.67	85.50	85.17	86.50	88.83
80	72.67	77.83	73.00	74.67	80.33
70	63.83	68.50	65.00	62.83	71.33
60	56.67	59.50	56.50	54.67	59.83
50	47.83	50.50	47.67	47.50	49.00

TABLE XXV

Percent Confidence Interval Coverage for System 2 Using the Univariate-Normal Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	88.67	91.17	93.50	96.17	97.67
95	83.00	83.83	85.67	90.67	92.17
90	77.17	79.17	81.33	85.83	88.00
80	68.67	71.00	73.50	75.67	78.33
70	61.33	63.50	63.33	65.00	66.00
60	54.00	53.50	55.50	55.17	56.17
50	44.67	46.17	45.67	44.50	47.33

TABLE XXVI

Percent Confidence Interval Coverage for System 3 Using the Univariate-Normal Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	79.17	88.17	89.83	94.17	95.33
95	67.33	75.67	77.33	84.17	90.83
90	59.17	66.33	68.17	75.17	84.33
80	45.50	53.17	53.33	63.17	71.33
70	35.83	42.67	43.67	49.17	59.67
60	26.83	33.83	34.50	42.67	47.33
50	18.83	24.33	26.33	32.83	35.50

TABLE XXVII

Percent Confidence Interval Coverage for System 4 Using the Univariate-Normal Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	89.50	90.50	93.67	96.17	97.50
95	83.33	84.50	86.67	90.83	92.83
90	77.50	78.67	82.00	85.67	88.83
80	68.67	71.17	72.83	76.17	77.17
70	61.00	61.83	64.33	65.17	66.17
60	53.33	53.33	54.83	54.83	55.67
50	46.00	45.00	44.67	44.33	47.83

TABLE XXVIII

Percent Confidence Interval Coverage for System 1 Using the Univariate-Normal Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	94.33	95.33	96.50	96.83	99.00
95	88.33	90.17	93.00	92.17	95.50
90	83.00	83.50	87.17	86.83	89.67
80	74.33	75.33	77.50	78.00	80.50
70	65.50	68.67	67.00	69.50	71.17
60	59.17	60.00	55.33	60.17	62.50
50	48.50	51.50	46.83	50.17	51.33

TABLE XXIX

Percent Confidence Interval Coverage for System 2 Using the Univariate-Normal Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	88.00	92.00	93.33	96.83	98.00
95	81.00	85.33	87.50	92.67	93.67
90	75.67	80.83	80.67	86.67	88.00
80	67.33	71.50	72.17	76.67	79.50
70	60.00	62.00	62.17	66.33	70.50
60	52.17	52.83	53.17	56.00	60.00
50	43.50	45.50	46.50	46.33	50.83

TABLE XXX

Percent Confidence Interval Coverage for System 3 Using the Univariate-Normal Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	80.67	86.67	90.17	94.50	98.00
95	69.00	75.00	79.00	86.83	92.00
90	59.83	66.33	71.17	79.33	83.33
80	46.83	53.50	53.33	65.83	72.17
70	35.83	43.83	41.33	55.00	60.83
60	26.50	33.50	31.17	43.33	50.17
50	19.17	23.67	23.33	35.50	39.83

TABLE XXXI

Percent Confidence Interval Coverage for System 4 Using the Univariate-Normal Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	87.67	91.50	93.17	97.00	97.83
95	81.17	85.83	87.33	92.83	93.50
90	75.83	80.50	81.17	86.00	88.67
80	67.17	71.17	71.67	77.00	78.33
70	60.50	62.50	62.33	66.83	70.00
60	51.50	52.33	54.00	56.33	60.67
50	42.17	45.33	46.33	47.17	50.67

TABLE XXXII

Percent Confidence Interval Coverage for System 1 Using the Univariate-Normal Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	96.67	96.33	96.33	98.00	98.83
95	91.00	89.50	91.67	94.67	95.17
90	86.00	85.33	86.33	90.50	90.00
80	77.67	75.50	77.00	78.17	82.00
70	66.83	65.67	67.00	67.33	71.00
60	56.83	56.50	57.50	59.00	61.50
50	47.00	47.17	48.17	48.50	53.50

TABLE XXXIII

Percent Confidence Interval Coverage for System 2 Using the Univariate-Normal Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	88.17	91.33	91.83	96.33	96.83
95	81.67	83.33	87.00	91.33	93.17
90	78.17	76.83	83.50	86.33	89.33
80	71.83	68.00	75.17	75.33	79.33
70	62.83	60.00	64.17	65.33	68.67
60	53.17	52.83	53.17	53.50	58.00
50	43.67	44.67	44.50	44.67	48.33

TABLE XXXIV

Percent Confidence Interval Coverage for System 3 Using the Univariate-Normal Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	80.67	86.67	89.33	96.00	97.83
95	67.67	73.50	80.00	87.33	91.17
90	58.83	65.50	70.33	80.83	84.17
80	46.17	52.83	57.83	66.33	72.50
70	34.83	41.33	47.00	54.83	61.17
60	25.17	31.50	34.83	42.33	50.83
50	18.50	21.83	23.00	32.17	43.17

TABLE XXXV

Percent Confidence Interval Coverage for System 4 Using the Univariate-Normal Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	89.17	90.83	91.83	96.33	97.00
95	83.17	83.33	87 00	92.33	93.00
90	78.83	77.33	82.67	86.83	88.83
80	73.00	68.50	75.83	75.67	79.17
70	61.50	60.33	62.83	64.50	68.83
60	54.33	53.67	53.50	54.17	59.83
50	45.00	44.83	44.00	44.17	48.17

various number of simulated reliability points, the percent confidence interval coverages by the univariate-normal technique are generally lower than the required confidence levels except for some cases involving component sample size 100. In these cases, the amount by which the coverage exceeds the required confidence level ranges from 0.33% to 3.50%. It should be noted that when simulating 200 system reliability points, there are only three occurrences of coverage exceeding the required confidence level (in System 1, for component sample size 15 and confidence level 50%; and for component sample size 100 and confidence levels 70% and 80%). As in the bivariate case, the highest coverage is obtained for System 1, the lowest coverage is obtained for System 3, and about equal coverage is obtained for Systems 2 and 4.

Tables XXXVI through LI present the percent confidence interval coverages from the univariate-beta technique for the four systems. The percent confidence interval coverages by the univariate-beta technique are always lower than the required confidence levels for System 3. For Systems 1, 2, and 4, the coverages are generally lower than the required confidence levels except for a few cases of component sample 100. It should be noted that the number of cases for which coverage exceeds a required confidence level is four when simulating 100 system reliability points and three when simulating 600 and 1000 system reliability points. In these cases, the amount by which the coverage exceeds a required confidence level is small (0.17% to 2.00%). When simulating 200 system reliability points, 12 cases of coverage exceeding

TABLE XXXVI

Percent Confidence Interval Coverage for System 1 Using the Univariate-Beta Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	96.83	97.83	98.67	98.33	98.67
95	91.50	94.00	92.83	93.67	93.83
90	84.17	88.33	87.67	89.00	88.33
80	72.00	76.17	76.33	77.83	79.17
70	63.00	68.00	66.50	68.17	70.33
60	52.67	56.67	56.50	57.33	59.33
50	42.83	48.33	46.33	47.17	49.50

TABLE XXXVII

Percent Confidence Interval Coverage for System 2 Using the Univariate-Beta Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	94.17	95.67	97.67	98.00	98.83
95	85.00	88.17	89.83	94.50	94.67
90	78.50	81.67	81.33	88.50	89.17
80	66.67	69.33	71.00	78.33	79.83
70	56.17	57.17	60.00	67.33	69.67
60	45.00	47.17	49.83	54.50	60.50
50	34.83	36.00	41.00	46.17	49.33

TABLE XXXVIII

Percent Confidence Interval Coverage for System 3 Using the Univariate-Beta Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	83.83	90.67	91.33	96.67	97.50
95	66.33	76.83	79.00	87.67	89.83
90	54.50	66.67	68.33	77.67	83.00
80	39.17	52.50	52.83	65.17	70.00
70	30.17	39.50	41.17	52.83	58.67
60	22.83	30.50	33.00	40.50	48.67
50	16.67	21.83	23.50	30.83	36.67

TABLE XXXIX

Percent Confidence Interval Coverage for System 4 Using the Univariate-Beta Technique and 100 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	94.17	95.83	97.17	97.83	98.50
95	84.17	87.33	89.67	94.17	93.50
90	78.00	82.33	82.33	88.00	89.00
80	66.00	68.50	70.17	77.67	79.83
70	56.50	56.67	59.17	66.67	69.67
60	46.83	46.67	49.67	55.00	60.33
50	36.00	35.50	41.00	46.50	50.17

TABLE XL

Percent Confidence Interval Coverage for System 1 Using the Univariate-Beta Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	98.00	97.50	98.50	98.83	98.83
95	92.17	92.83	92.17	94.33	94.83
90	86.83	85.83	87.50	90.33	90.67
80	75.83	74.83	77.67	81.33	80.33
70	64.17	69.17	67.83	70.83	69.17
60	55.17	60.17	56.67	62.17	58.50
50	44.83	49.00	46.67	51.83	48.17

TABLE XLI

Percent Confidence Interval Coverage for System 2 Using the Univariate-Beta Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	92.50	96.00	95.83	98.17	98.33
95	84.83	88.00	90.17	93.67	94.67
90	77.67	81.83	83.17	87.67	90.67
80	65.67	67.83	71.50	76.00	82.50
70	55.17	58.67	60.83	64.50	71.00
60	45.67	48.33	50.67	53.00	61.17
50	37.67	41.00	41.00	46.17	50.50

TABLE XLII

Percent Confidence Interval Coverage for System 3 Using the Univariate-Beta Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	84.50	90.17	92.83	96.83	97.50
95	64.00	75.00	80.33	90.67	89.67
90	54.00	65.50	69.67	82.67	82.67
80	39.67	52.17	54.33	68.50	70.17
70	28.33	41.67	41.00	57.17	56.17
60	21.83	30.83	29.67	45.17	48.00
50	15.00	22.83	21.83	36.17	38.50

TABLE XLIII

Percent Confidence Interval Coverage for System 4 Using the Univariate-Beta Technique and 200 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	92.00	95.83	95.83	98.17	98.17
95	84.67	87.83	90.67	93.50	94.67
90	77.17	82.17	82.83	88.17	90.83
80	66.00	68.50	72.17	75.83	81.83
70	55.67	58.33	61.67	64.83	72.00
60	45.17	49.17	50.67	52.17	61.33
50	37.83	41.00	41.17	45.50	50.83

TABLE XLIV

Percent Confidence Interval Coverage for System 1 Using the Univariate-Beta Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	97.33	98.33	98.33	98.67	99.00
95	92.00	93.00	92.83	93.67	95.50
90	83.67	85.83	86.33	88.17	88.67
80	74.33	75.17	75.83	78.33	77.00
70	61.00	65.50	65.00	69.67	69.33
60	52.50	55.50	55.33	58.67	62.00
50	41.83	45.33	46.33	48.67	51.50

TABLE XLV

Percent Confidence Interval Coverage for System 2 Using the Univariate-Beta Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	92.33	95.50	96.50	97.83	98.33
95	84.83	89.83	89.00	92.17	93.83
90	78.67	82.83	82.00	87.17	88.33
80	66.00	70.83	71.50	75.17	77.67
70	53.83	58.67	61.83	64.00	67.33
60	47.17	47.50	51.50	53.50	56.83
50	37.83	38.67	41.83	44.00	48.67

TABLE XLVI

Percent Confidence Interval Coverage for System 3 Using the Univariate-Beta Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	80.50	89.50	91.33	97.00	97.50
95	63.67	74.83	77.00	87.83	90.83
90	52.17	63.83	66.83	80.33	82.83
80	38.67	47.33	53.00	67.83	69.50
70	29.17	38.17	43.00	55.33	61.00
60	22.50	27.50	33.33	43.33	50.33
50	14.17	20.33	21.83	32.33	38.83

TABLE XLVII

Percent Confidence Interval Coverage for System 4 Using the Univariate-Beta Technique and 600 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	92.33	95.17	96.33	97.83	98.50
95	85.50	89.83	89.50	92.83	94.33
90	78.83	83.00	82.00	87.33	89.33
80	66.50	69.83	71.50	75.00	77.00
70	54.50	58.17	61.17	64.50	66.83
60	46.17	49.00	51.33	55.00	56.67
50	38.33	38.00	41.67	43.67	48.17

TABLE XLVIII

Percent Confidence Interval Coverage for System 1 Using the Univariate-Beta Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	96.83	98.17	98.33	99.00	99.00
95	90.00	93.50	94.00	94.00	94.50
90	82.67	87.17	88.50	89.17	89.00
80	71.33	77.00	78.17	77.83	78.00
70	59.83	65.50	68.83	66.67	69.17
60	50.50	55.67	56.83	57.67	60.17
50	40.50	44.83	44.67	48.33	51.50

TABLE XLIX

Percent Confidence Interval Coverage for System 2 Using the Univariate-Beta Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	95.83	94.83	96.33	98.33	97.83
95	86.33	87.50	90.17	93.50	91.67
90	78.67	79.83	85.67	87.67	84.83
80	64.50	69.83	73.67	75.67	77.00
70	54.17	59.33	62.00	63.00	67.17
60	45.50	48.33	51.33	53.33	60.17
50	37.50	42.17	40.50	41.83	49.00

TABLE L

Percent Confidence Interval Coverage for System 3 Using the Univariate-Beta Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	81.00	90.17	92.17	96.50	97.83
95	63.33	75.67	80.83	87.67	89.67
90	51.83	63.50	70.67	78.50	83.33
80	35.67	47.67	55.67	63.00	69.67
70	26.33	36.67	40.83	53.33	60.00
60	20.50	28.83	31.17	43.00	48.83
50	14.00	22.67	24.00	34.17	39.67

TABLE LI

Percent Confidence Interval Coverage for System 4 Using the Univariate-Beta Technique and 1000 Point Estimates

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	95.67	95.17	96.00	98.17	97.67
95	86.50	87.17	90.17	93.83	91.50
90	79.00	80.50	85.83	87.83	85.33
80	64.83	69.00	72.83	75.33	75.67
70	53.67	59.00	62.33	63.33	68.00
60	45.00	49.33	51.33	53.17	58.00
50	37.00	42.00	40.33	41.67	48.17

a required confidence level occur for the sample size of 100 and 5 cases occur for the sample size of 50 (these 5 are all in System 1). As in the bivariate and univariate-normal cases, the most coverage by the univariate-beta technique is obtained for System 1, the least coverage is obtained for System 3, and approximately equal coverage is obtained for System 2 and 4.

For all three techniques, two patterns regarding coverage can be found in the tabled results. The first pattern is that as the component sample size increases for a required confidence level in any system, the percent confidence interval coverage converges toward the required confidence level. This is a reasonable result since as the component sample size increases, the technique has more information on which to base an estimate of the system reliability. In addition, as the sample size increases, the distributions assumed to be normal in these techniques become more and more like true normal distributions. This could also account for the exceptions in coverage patterns found in all three techniques that occur in cases involving the highest component sample size of 100.

The second pattern is that the percent confidence interval coverage is inversely related to the true system reliability. System 3, which has the highest true system reliability (.99841), received the lowest coverage. Systems 2 and 4 have approximately equal system reliabilities (.84297 and .84197 respectively) and also received approximately equal coverage. System 1, which has the lowest true

system reliability (.68448), received the highest coverage. Thus, as the true system reliability decreases, the coverage provided by any of these three techniques increases.

Comparison of Techniques

When compared to the univariate-normal technique, the bivariate technique generally results in a higher percent coverage of the true system reliability except in the cases where the component sample size is 100 and the required confidence levels are 50%, 60%, and 70%. However, in these cases, the univariate-normal technique generally results only in 1% to 3% more coverage. It is of interest to note that when simulating 200 system reliability points, the bivariate case results in higher coverage for virtually all component sample sizes including the component sample size of 100 (the one exception is in System 4 at the 50% confidence level for component sample size 10).

When compared to the univariate-beta technique, the bivariate technique results in a higher percent coverage of the true system reliability with some exceptions. The major exceptions are the coverages for component sample size 100 when simulating 200, 600, and 1000 system reliability points. Also, when simulating 100 system reliability points for the cases of component sample size 50, Systems 2 and 4 show a higher coverage by the univariate-beta technique for the 50 to 80 percent confidence levels. The coverage by the univariate-beta technique is better by 0.33% to 5.00% in these cases. As the number of simulated system reliability

points increases from 100 to 1000, the number of times the univariate-beta technique has higher coverage decreases.

In comparing the univariate-normal and univariate-beta techniques, the univariate-beta technique is generally better at the higher required confidence levels, i.e., at the 90, 95, and 99 percent confidence levels. However, as the number of system reliability points generated increases to 1000, the univariate-normal technique becomes better for the component sample size of 100 for all four systems and for the component sample size of 10 for Systems 1 and 3. It should be noted that the coverage provided by the univariate-beta technique when generating 200 system reliability points is much better for Systems 1 and 3 than when 100, 600, and 1000 system reliability points are generated.

Theoretically, as the number of system reliability points generated increases, the percent coverage of the true system reliability should converge to the required confidence level. Thus, when using one of these techniques to estimate a system's reliability, 1000 system reliability points should be simulated. However, approximately the same amount of coverage was obtained when generating 100, 200, 600, and 1000 points. Thus, when conducting accuracy tests of these techniques using Monte Carlo simulation, 100 points instead of higher numbers of points could be used to examine the effect of any modifications to the techniques since the computer execution times for these techniques are quite high (see Appendices B, C, and D).

Results of Gatliffe's Technique

The confidence interval coverage results of the univariate-normal technique after being adjusted for perfect system reliability estimates are listed in Tables LII through LV. The result of applying Gatliffe's method to the univariate-normal technique is that there is no change in the percent confidence interval coverage for Systems 1, 2, and 4. For System 3, the coverage obtained by applying Gatliffe's method is increasingly better for component sample sizes 20, 15, and 10 respectively. The improvement ranges from an increase of 4.5% to 60.0% coverage. There is very little improvement for component sample size 50 and no improvement for the component sample size 100.

When compared to the univariate-beta technique, this adjusted univariate-normal technique is still better for component sample sizes 10, 15, and 20 in System 3 and slightly better for component sample size 100 in System 3. When compared to the bivariate technique, better coverage is obtained with this technique for component sample sizes 10 and 15 and for component sample size 20 at the 80%, 90%, 95%, and 99% confidence levels.

The result of significant improvement for only System 3 was expected as discussed in Section III. However, no improvement at all for the other systems should have been expected since all system reliability estimates used by Gatliffe's method to replace perfect system reliability

TABLE LII

Percent Confidence Interval Coverage for System 1 Using the Adjusted Univariate-Normal Technique

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	94.33	95.33	96.50	96.83	99.00
95	88.33	90.17	93.00	92.17	95.50
90	83.00	83.50	87.17	86.83	89.67
80	74.33	75.33	77.50	78.00	80.50
70	65.50	68.67	67.00	69.50	71.17
60	59.17	60.00	55.33	60.17	62.50
50	48.50	51.50	46.83	50.17	51.33

TABLE LIII

Percent Confidence Interval Coverage for System 2 Using the Adjusted Univariate-Normal Technique

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	88.00	92.00	93.33	96.83	98.00
95	81.00	85.33	87.50	92.67	93.67
90	75.67	80.83	80.67	86.67	88.00
80	67.33	71.50	72.17	76.67	79.50
70	60.00	62.00	62.17	66.33	70.50
60	52.17	52.83	53.17	56.00	60.00
50	43.50	45.50	46.50	46.33	50.83

TABLE LIV

Percent Confidence Interval Coverage for System 3 Using the Adjusted Univariate-Normal Technique

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	100.00	100.00	100.00	98.17	98.00
95	100.00	100.00	100.00	87.67	92.00
90	100.00	100.00	100.00	79.83	83.33
80	100.00	95.50	69.00	66.17	72.17
70	95.00	60.33	49.67	55.17	60.83
60	56.00	45.17	37.17	43.33	50.17
50	36.33	33.50	27.83	35.50	39.83

TABLE LV

Percent Confidence Interval Coverage for System 4 Using the Adjusted Univariate-Normal Technique

Required Confidence Level (Percent)	Sample Size 10	15	20	50	100
99	87.67	91.50	93.17	97.00	97.83
95	81.17	85.83	87.33	92.83	93.50
90	75.83	80.50	81.17	86.00	88.67
80	67.17	71.17	71.67	77.00	78.33
70	60.50	62.50	62.33	66.83	70.00
60	51.50	52.33	54.00	56.33	60.67
50	42.17	45.33	46.33	47.17	50.67

estimates are high in value, i.e., above 0.95 in value (see Table III). Since System 3 has the only true reliability greater than 0.95, it is the only system that can have an increase in the number of system reliability estimates that are less than or equal to the true system reliability during the Monte Carlo simulations. Thus, the adjusted univariate-normal technique will only be more effective in predicting the system reliability than the other techniques if the true reliability is believed to be larger than 0.95 and if the reliability is being estimated for a component sample size of 20 or less.

Significant improvement in the percent confidence interval coverage was only noticed for component sample sizes 20, 15, and 10 because of the percentage of the component reliability estimates generated with values of one (Step 5 of the univariate-normal technique). The percentages are as follows:

Component Sample Size	Percentage
10	8.08
15	4.40
20	2.24
50	0.093
100	0.009

As can be seen, the number of component reliability estimates of value one is quite small for component sample sizes 50 and 100 and increasingly larger for sample sizes 20, 15, and 10. This is due to the fact that the distribution of the component reliability estimates is normal for component sample sizes 50 and 100 but not for component sample sizes 20, 15,

and 10 (see Section II, page 57). Recall that in System 3, only one perfect component reliability estimate is needed to obtain a perfect system reliability estimate. Because the number of perfect component reliability estimates increases as the sample size decreases, there are increasingly more chances to have perfect system reliability estimates--and thus apply Gatliffe's technique--as the sample size decreases. Thus, the coverage for the adjusted univariate-normal technique is increasingly better as the component sample size decreases from 20 to 10.

Conclusions

In summary, the following five conclusions are drawn from the data for the bivariate, univariate-normal, and univariate-beta Monte Carlo simulation techniques:

1. For any given technique, accuracy varies indirectly with the true system reliability. As the true system reliability decreases, the accuracy of the technique increases. As the true system reliability increases, the accuracy of the technique decreases.

2. The bivariate technique is the most accurate of the three techniques regardless of the number of system reliability points being generated.

3. The univariate-beta technique is generally more accurate than the univariate-normal technique at the higher required confidence levels.

4. Gatliffe's method of adjusting for perfect system reliability will only improve the accuracy of the univariate-

normal technique for component sample sizes of 20 or less if the true system reliability is greater than 0.95. Thus, the adjusted univariate-normal technique should be used to estimate a system's reliability only if there is reason to believe it is higher than 0.95.

5. The accuracy of each technique does not vary greatly as the number of system reliability points simulated changes. A tradeoff between computer execution time and resulting accuracies needs to be made by the user of the technique.

Recommendations

Under the testing conditions used in this thesis, if desiring to estimate a systems reliability, use:

1. The bivariate technique if the true system reliability is believed to be below 0.95.

2. The adjusted univariate-normal technique if the true system reliability is believed to be greater than or equal to 0.95 and the component data has a sample size of 20 or less.

Further work should be done to improve the accuracy of the bivariate technique. In such a case, the code should be restructured to decrease the execution time required to run the program.

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A. Derivation of Gatliffe's Failure Equation

In deriving Eq (26), Gatliffe began by examining the expression for a system's reliability under the condition that the system network be viewed as a set of components or subassemblies connected in logical series. The reliability of a system with k components or subassemblies in series is

$$Rs = \prod_{i=1}^{k} P_i \tag{35}$$

where P_i is the reliability of the ith component or subassembly. Let

$$s = -\ln Rs \tag{36}$$

$$= -\sum_{i=1}^{k} \ln(1-Q_i) \quad \text{where} \quad Q_i = 1-P_i$$

$$= \sum_{i=1}^{k} \left(Q_i + \frac{Q_i^2}{2} + \frac{Q_i^3}{3} + \ldots\right)$$

using the series expansion for $\ln(1-x)$, i.e., $\ln(1-x) = -\sum_{j=1}^{\infty} \frac{(x)^j}{j}$ if $0 \le x \le 1$. For small values of Q_i, i.e., $P_i \ge .9$,

$$s \approx \sum_{i=1}^{k} \left(Q_i + \frac{Q_i^2}{2}\right)$$

with maximum error of 0.34% of true value (Ref 9:10).

Letting $T_i = Q_i + \frac{Q_i^2}{2}$,

$$s = \sum_{i=1}^{k} T_i \tag{37}$$

Gatliffe showed that $\hat{T}_i$ can be written in general as $\hat{T}_i = A_i\hat{Q}_i + \frac{B_i}{2}\hat{Q}_i^2$ where A_i and B_i are chosen so as to make T_i unbiased. Thus $A_i = \frac{(2N_i-3)}{(2N_i-2)}$ and $B_i = \frac{N_i}{N_i-1}$ where N_i is the number of tests on the ith component whose failures F_i are binomially distributed with parameters Q_i and N_i ; $Q_i = \frac{F_i}{N_i}$ (Ref 9:38-39). He thus showed that $\hat{T}_i$ is an unbiased estimator of T_i and that $\hat{s} = \sum_{i=1}^{k} \hat{T}_i$ is therefore an unbiased estimator of s (Ref 9: 38-39). He also showed for component "unreliability" Q_i and component sample size N_i (Ref 9:40-43) that

$$\begin{aligned} \mathrm{Var}(\hat{s}) &\simeq \sum_{i=1}^{k} \mathrm{Var}(\hat{T}_i) \\ &= \sum_{i=1}^{k} \left(\frac{T_i}{N_i}\right) \end{aligned} \tag{38}$$

Gatliffe intuitively assumed that the probability distribution of $\hat{s}$ is gamma with parameters r and θ , i.e.,

$$f_{\hat{s}}(x,r,\theta) = \begin{cases} \frac{1}{\Gamma(r)\theta^r} x^{r-1} e^{-x/\theta} & , \quad x \geq 0 \quad \text{and} \quad \theta > 0 \\ 0 & , \quad \text{otherwise} \end{cases} \qquad (39)$$

with mean $E(\hat{s}) = r\theta = \sum_{i=1}^{k} T_i$ and variance $Var(\hat{s}) = r\theta^2$

$= \sum_{i=1}^{k} (\frac{T_i}{N_i})$ (Ref 9:11). Using these expressions for the mean and variance of $\hat{s}$, Gatliffe solved these expressions for r and θ (Ref 9:12):

$$r = \frac{(\sum_{i=1}^{k} T_i)^2}{\sum_{i=1}^{k} \frac{T_i}{N_i}} \qquad (40)$$

$$\theta = \frac{\sum_{i=1}^{k} \frac{T_i}{N_i}}{\sum_{i=1}^{k} T_i} \qquad (41)$$

since $\sum_{i=1}^{k} T_i = -\ln Rs$ and $E(\hat{s}) = r\theta$, the parameter θ can be rewritten as $\theta = \frac{r\theta}{r}$ and thus

$$\theta = \frac{E(s)}{r} = \frac{-\ln Rs}{r} \qquad (42)$$

Since $\hat{s}$ has a gamma $(r,\hat{\theta})$ distribution, $\frac{2\hat{s}}{\theta}$ has a $\chi^2_{(2r)}$ distribution. This means that for a given significance level α,

$$1-\alpha = P[\frac{2\hat{s}}{\theta} \geq \chi^2_{(2r,1-\alpha)}]$$

Thus,

$$1-\alpha = P[\theta \leq \frac{2\hat{s}}{\chi^2_{(2r,1-\alpha)}}]$$

Using the fact that $\theta = \frac{-\ln Rs}{r}$,

$$1-\alpha = P[\frac{-\ln Rs}{r} < \frac{2\hat{s}}{\chi^2_{(2r,1-\alpha)}}]$$

$$1-\alpha = P[\ln Rs \geq \frac{-2r\hat{s}}{\chi^2_{(2r,1-\alpha)}}]$$

Thus, $Rs^*(\alpha)$, the $100(1-\alpha)\%$ lower confidence limit for Rs can be expressed as (Ref 9:12-13):

$$Rs^*(\alpha) = \exp(\frac{-2r\hat{s}}{\chi^2_{(2r,1-\alpha)}}) \tag{43}$$

The $100(1-\alpha)\%$ lower confidence limit estimate for Rs is then just

$$\hat{Rs}^*(\alpha) = \exp(\frac{-[2\hat{r}]\hat{s}}{\chi^2_{([2\hat{r}],1-\alpha)}}) \tag{44}$$

where $2\hat{r}$ is replaced with $[2\hat{r}]$ since it is very difficult to calculate chi-square values of noninteger degrees of freedom. Gatliffe justified this substitution by showing that the ratio of the chi-square value to its number of

degrees of freedom changes slowly as the number of degrees of freedom changes. He also used computer routines to calculate some chi-square values with noninteger degrees of freedom and found that a difference in the chi-square value is, in the worst case, not noticed until the third decimal place (Ref 9:13).

When all components have zero failures, $\hat{s}$ will have a value of zero and Eq (44) will yield a lower confidence limit estimate, $\hat{R}s^*(\alpha)$, of one. The situation is unrealistic because components do fail.

Gatliffe stated that conducting an equal number of tests, N, without failure on all the series system components and subassemblies is the same as conducting N tests on the system without failure (Ref 9:44). Thus, the number of system successes, W, can be related to Rs by the following binomial formula:

$$\sum_{j=W}^{N} \binom{N}{j} Rs^j (1-Rs)^{N-j} = \text{Prob (number successes} \geq W)$$

The $100(1-\alpha)\%$ lower confidence limit estimate for system reliability is thus the solution Rs^* to

$$\sum_{j=W}^{N} \binom{N}{j} Rs^{*j} (1-Rs^*)^{N-j} = \alpha$$

When there are no system failures, $W=N$ and the above equation collapses to

$$(Rs^*)^N = \alpha$$

$$Rs^* = (\alpha)^{1/N} \tag{45}$$

Equating this result to Eq (44) yields

$$(\alpha)^{1/N} = \exp\left(\frac{[-2\hat{r}]\hat{s}}{\chi^2_{([2\hat{r}],1-\alpha)}}\right) \tag{46}$$

from which

$$\hat{s} = \frac{\chi^2_{([2\hat{r}],1-\alpha)}}{N[2\hat{r}]} (\ln \alpha) \tag{47}$$

$$\hat{s} = \frac{\chi^2_{(2,1-\alpha)}}{2N} (\ln\alpha) \tag{48}$$

since for any N_i , $\hat{r}=1$ because $\hat{r} = max[1,r]$

$= \max \left[1.0, \ \frac{(\sum_{i=1}^{k} \hat{T}_i)^2}{\sum_{i=1}^{k} \frac{\hat{T}_i}{N_i}} \right]$. This reduces to

$\hat{r} = \max[1.0, N_i\hat{T}_i]$ and $\hat{s} = \hat{T}_i$ because only one component or subassembly will be adjusted to make a nonzero contribution to the sums $\sum_{i=1}^{k} \hat{T}_i$ and $\sum_{i=1}^{k} \frac{T_i}{N_i}$ (Ref 9:45).

If $\hat{r}=1.0$, then $\hat{T}_i N_i \leq 1$. This implies that

$$\hat{T}_i \leq \frac{1}{N_i}$$

Since $\hat{T}_i$ can be expressed as $A_i\hat{Q}_i + \frac{B_i}{2}\hat{Q}_i^2$,

$$A_i\hat{Q}_i + \frac{B_i}{2}\hat{Q}_i^2 \leq \frac{1}{N_i}$$

Using the fact that $A_i = \frac{2N_i-3}{2(N_i-1)}$, $B_i = \frac{N_i}{N_i-1}$, and

$\hat{Q}_i = \frac{F_i}{N_i}$, the equation becomes

$$F_i^2 + (2N_i-3)F_i - 2(N_i-1) \leq 0 \qquad (49)$$

Re-expressing the above equation as

$$(F_i-1)(F_i+2(N_i-1)) \leq 0 \qquad (50)$$

one can see that the solution is some F_i belonging to the interval $(0,1]$.

Since $\hat{s} = T_i = A_iQ_i + \frac{B_i}{2}Q_i^2$,

$$\hat{s} = A_i\frac{F_i^*}{N} + \frac{B_i(F_i^*)^2}{2N^2} \qquad (51)$$

Since Eqs (48) and (51) equal $\hat{s}$, they can be equated as follows:

$$\frac{\chi^2_{(2,1-\alpha)}}{2N}(\ln \alpha) = \frac{A_iF_i^*}{N} + \frac{B_i(F_i^*)^2}{2N^2} \qquad (52)$$

Rearranging terms yields the following

$$(F_i^*)^2 + (2N-3)F_i^* + (N-1) \ln \alpha(\chi^2_{(2,1-\alpha)}) = 0$$

which is Eq (26). Solving for F_i^* as a function of N and α yields the failures with which F_i can be replaced if $F_i=0$ in the expression $P_i = 1 - \frac{F_i}{N_i}$ (Ref 9:45-46).

B. Bivariate Technique: Notes and Computer Program Listings

This appendix contains the computer program listings for the bivariate technique plus the definitions of major variables in the main program SYSRE. In addition, Table LVI lists the execution times of the bivariate technique for the various numbers of simulated reliability points and component sample sizes. The execution times are the number of seconds needed to perform one computer run of the bivariate technique (120,000 words of core) on a Control Data Corporation (CDC) 6600 machine. It should be noted that all 600 Monte Carlo simulations for a particular component sample size were performed in one computer run only for 99 and 199 simulated reliability points. For the cases involving 599 simulated reliability points, six computer runs of 100 Monte Carlo simulations each were performed due to the excessive processing time needed for one computer run of 600 Monte Carlo simulations. The execution times for 100 Monte Carlo simulations only are thus listed in the table. Similarly, the execution times for 50 Monte Carlo simulations per computer run are listed for the various component sample sizes when simulating 999 system reliability points.

TABLE LVI

Bivariate Technique: Execution Times (Seconds)

Number Simulated Reliability Points	Component Sample Size 10	15	20	50	100
99 600 simulations per run	584	651	725	1201	1877
199 600 simulations per run	1469	1543	1600	1800	2138
599 100 simulations per run	955	960	977	1044	1175
999 50 simulations per run	1251	1260	1275	1294	1373

List of Major Variables for Program SYSRE

AA = number of samples censored from below for component k

BB = quantity M such that N-M samples are censored from above for component k

CC = sample size of component k before censoring ($N \leq 500$)

C99(L)
C95(L)
C90(L)
C80(L)
C70(L)
C60(L)
C50(L)
= for L = system 1, 2, 3, or 4 , the number of times the calculated xx percent confidence interval covers the true system reliability, where xx are the two digits in Cxx(L)

ET(1),ETH(K) = estimate of the scale parameter for component k

EKK(1),EEK(K) = estimate of the shape parameter for component k

TC(1),TXC(K) = estimate of the location parameter for component k

EK1 = sample estimate of scale parameter

TH1 = sample estimate of shape parameter

EK(K) = for component k , the maximum likelihood estimate of the shape parameter

TH(K) = for component k , the maximum likelihood estimate of the scale parameter

LCOUNT(K) = counter for truncation of scale parameter for component k

K,NCOMP = number of system components

NCOUNT(K) = counter for number of component reliability estimates greater than or equal to one for component k

NRUNS = number of Monte Carlo simulations desired

NUM = number of system reliability points to be simulated

MABOV(K) = quantity M such that N-M samples are censored from above for component k

MRLOW(K) = number samples censored from below for component k

NSAMP(K) = sample size of component k before censoring ($N \leq 500$)

R(K) = reliability estimate for component k

RC(K) = true reliability for component k

RS(2000,L) = array of system reliability estimates for system L , L=1,2,3,4

RUNS = number Monte Carlo simulations desired

Q1 = percent component sample size censored from below

Q2 = percent component sample size censored from above

S11 = element (1,1) of the information matrix

S12 = elements (1,2) and (2,1) of the information matrix

S22 = element (2,2) of the information matrix

SB = standard deviation for estimating the shape parameter

ST = standard deviation for estimating the scale parameter

T(500,K) = array of random failure times from the Weibull distribution for component k

TIME = mission time

TRS(4) = true system reliability for systems 1, 2, 3, and 4

V(K,2,2) = for component k , the array of four standard deviation elements of the shape and scale parameter estimates from the 2x2 variance-covariance matrix

V11 = element (1,1) of the variance-covariance matrix

V12 = elements (1,2) and (2,1) of the variance-covariance matrix

V22 = element (2,2) of the variance-covariance matrix

```
      PROGRAM SYSRE(INPUT,OUTPUT)
C*****NCOMP=NUMBER OF COMPONENTS IN SYSTEM
C*****NUM=NUMBER OF SIMULATIONS DESIRED
C*****TIME=MISSION TIME
C*****EEK=ESTIMATE OF SHAPE PARAMETER FOR THAT COMPONENT
C*****ETH=ESTIMATE OF SCALE PARAMETER FOR THAT COMPONENT
C*****XC=LOCATION PARAMETER FOR THAT COMPONENT
C*****NSAMP=SAMPLE SIZE BEFORE CENSORING(N=100 OR LESS)
C*****MABOV=QUANTITY M WHERE N-M SAMPLES CENSORED FROM ABOVE
C*****MRLOW=NUMBER OF SAMPLES CENSORED FROM BELOW
      DIMENSION TC(550),ET(550),EKK(550),MSIZE(5)
      DIMENSION S(2,2),V(10,2,2),R(10),NSAMP(10),
     CMABOV(10),MRLOW(10),EEK(10),ETH(10),TXC(10),TH(10),EK(10)
      DIMENSION TRS(4),RC(5),RS(2000,4),C99(4),C95(4),
     CC90(4),C80(4),C70(4),C60(4),C50(4)
      DIMENSION T(500,10)
      DIMENSION NCOUNT(5)
      DIMENSION LCOUNT(5)
      DATA LCOUNT/0,0,0,0,0/
      DATA NCOUNT /0,0,0,0,0/
      READ*,RUNS,NRUNS
      READ 50,NCOMP,NUM,TIME
   50 FORMAT(I5,I5,F20.8)
      PRINT 55
   55 FORMAT(1H ,17HNO. OF COMPONENTS,5X,18HNO. OF SIMULATIONS.
     C      5X,12HMISSION TIME)
      PRINT 60,NCOMP,NUM,TIME
   60 FORMAT(1H ,7X,I5,17X,I5,15X,F20.8)
      CALL CLOCK(AA)
      CALL RANSET(AA)
      DO 25 L=1,4
      C99(L)=0.0
      C95(L)=0.0
      C90(L)=0.0
      C80(L)=0.0
      C70(L)=0.0
      C60(L)=0.0
      C50(L)=0.0
   25 CONTINUE
      DO 550 N=1,NRUNS
      DO 132 K=1,NCOMP
      IF(NX.NE.1) GO TO 66
      READ 70,NSAMP(K),MABOV(K),MRLOW(K),ETH(K),
     CEEK(K),TXC(K)
   70 FORMAT(3I3,3F8.4)
      RC(K)=EXP(-(((TIME-TXC(K))/ETH(K))**EEK(K)))
      PRINT 72,K,ETH(K),EEK(K),TXC(K)
   72 FORMAT(1H ,9HCOMPONENT,I3,5X,6HSCALE=,F8.4,5X,6HSHAPE=,
     CF8.4,5X,9HLOCATION=,F8.4)
   66 CALL RANEI(EEK(K),ETH(K),TXC(K),T(1,K),NSAMP(K),K)
      TC(1)=TXC(K)
      ET(1)=ETH(K)
      EKK(1)=EEK(K)
      CALL PARES(NSAMP(K),MABOV(K),TC,ET,
     CEKK,MRLOW,TH(K),EK(K),T(1,K),NX)
      IF(TH(K).LE.0.0) PRINT 2
    2 FORMAT(1H ,13HTHETA LE ZERO)
```

```
      IF(TH(K).LE.0.) GO TO 240
      AA=MRLOH(K)
      BB=MADOJ(K)
      CC=NSAMP(K)
      Q1=AA/CC
      Q2=(CC-BB)/CC
      CALL VACCV(Q1,Q2,EK(K),CC,TH(K),V11,V12,V22)
      A=V11
      B=V12
      C=V22
      IF (V12)110,100,110
  100 V(K,1,1)=SQRT(V11)
      V(K,1,2)=0.
      V(K,2,1)=0.
      V(K,2,2)=SQRT(V22)
      V(1,2)=0.
      V(2,1)=0.
      V(2,2)=SQRT(V22)
      GO TO 132
  110 Y=.5*SQRT(A*A-2.*A*C+C*C+4.*B*B)
      X1=.5*(A+C)+Y
      X2=.5*(A+C)-Y
      R1=SQRT(1.+(X1-A)**2/B**2)
      R2=SQRT(1.+(X2-C)**2/B**2)
      S(1,1)=1./R1
      S(2,2)=1./R2
      S(1,2)=(X2-C)/(B*R2)
      S(2,1)=(X1-A)/(B*R1)
      X1=SQRT(X1)
      X2=SQRT(X2)
      D1=S(1,1)*X1
      D2=S(1,2)*X2
      D3=S(2,1)*X1
      D4=S(2,2)*X2
      V(K,1,1)=D1*S(1,1)+D2*S(1,2)
      V(K,1,2)=D1*S(2,1)+D2*S(2,2)
      V(K,2,1)=D3*S(1,1)+D4*S(1,2)
      V(K,2,2)=D3*S(2,1)+D4*S(2,2)
  132 CONTINUE
C*****CALCULATE TRUE SYSTEM RELIABILITIES
      TRS(1)=RC(1)*RC(2)*RC(3)
      TRS(2)=RC(1)*(1.0-(1.0-RC(2))*(1.0-RC(3)))
      TRS(3)=1.0-(1.0-RC(1))*(1.0-RC(2))*(1.0-RC(3))
      TRS(4)=RC(1)*(1.0-(1.0-RC(2))*(1.0-RC(5)*(1.0-(1.0-RC(3))*(1.0-RC(
     C4)))))
      IF(NX.NE.1) GO TO 133
      DO 140 JJ=1,4
      PRINT 141,JJ,TRS(JJ)
  141 FORMAT(1H ,27HTRUE RELIABILITY FOR SYSTEM,I2,2H=,F8.6)
  140 CONTINUE
C*************GENERATION OF SAMPLE VALUE OF PARAMETERS
  133 DO 170 J=1,NUM
      DO 150 K=1,NCOMP
      W=RANOR(X)
      B=RANOR(X)
      ST=V(K,1,1)*W+V(K,1,2)*B
      SB=V(K,1,2)*W+V(K,2,2)*B
```

```
      TH1=STH+TH(K)
      EK1=SEK+EK(K)
C*****GENERATING SAMPLE WEIBULL COMPONENT RELIABILITIES
      IF(TH1.LE.0.0) LCOUNT(K)=LCOUNT(K)+1
      IF(TH1.LE.0.0) R(K)=0.0
      IF(TH1.LE.0.0) GO TO 148
      ARGUM=-((TIME-TXC(K))/TH1)**EK1
      IF(ARGUM.GT.-20.0.AND.ARGUM.LT.0.0) GO TO 145
      IF(ARGUM.GE.0.0) R(K)=1.0
      IF(ARGUM.LE.-20.0) R(K)=0.0
      GO TO 148
  145 R(K)=EXP(ARGUM)
  148 IF(R(K).GE.1.0) NCOUNT(K)=NCOUNT(K)+1
  150 CONTINUE
C*****GENERATING SAMPLE SYSTEM RELIABILITY
      RS(J,1)=R(1)*R(2)*R(3)
      RS(J,2)=R(1)*(1.0-(1.0-R(2))*(1.0-R(3)))
      RS(J,3)=1.0-(1.0-R(1))*(1.0-R(2))*(1.0-R(3))
      RS(J,4)=R(1)*(1.0-(1.0-R(2))*(1.0-R(5)*(1.0-(1.0-R(3))*(1.0-R(4)))
     C))
  170 CONTINUE
C*****ORDERING SAMPLE SYSTEM RELIABILITIES
      DO 205 L=1,4
      ML=1
  175 MH=NUM-1
  176 MHT=MH
      ISW=1
      DO 200 I=ML,MHT
      II=I+1
      IF(RS(I,L)-RS(II,L)) 200,200,180
  180 GO TO (185,190),ISW
  185 ISW=2
      IF(I-1)190,190,187
  187 ML=I-1
  190 TEMP=RS(I,L)
      RS(I,L)=RS(II,L)
      RS(II,L)=TEMP
      MH=I-1
  200 CONTINUE
      GO TO (205,176),ISW
  205 CONTINUE
      IF(NX.NE.1) GO TO 500
      READ 320,M1,M2,M3,M4,M5,M6,M7
  320 FORMAT(7I4)
  500 DO 510 L=1,4
      IF(TRS(L).GE.RS(M1,L)) C99(L)=C99(L)+1.0
      IF(TRS(L).GE.RS(M2,L)) C95(L)=C95(L)+1.0
      IF(TRS(L).GE.RS(M3,L)) C90(L)=C90(L)+1.0
      IF(TRS(L).GE.RS(M4,L)) C80(L)=C80(L)+1.0
      IF(TRS(L).GE.RS(M5,L)) C70(L)=C70(L)+1.0
      IF(TRS(L).GE.RS(M6,L)) C60(L)=C60(L)+1.0
      IF(TRS(L).GE.RS(M7,L)) C50(L)=C50(L)+1.0
  510 CONTINUE
  550 CONTINUE
  600 DO 610 L=1,4
      C99(L)=C99(L)/SUMS
      C95(L)=C95(L)/SUMS
```

```
      C90(L)=C99(L)/RUNS
      C80(L)=C80(L)/RUNS
      C70(L)=C70(L)/RUNS
      C60(L)=C60(L)/RUNS
      C50(L)=C50(L)/RUNS
610   CONTINUE
      DO 625 L=1,4
      PRINT 620,L,C99(L),C95(L),C90(L),C80(L),C70(L),C60(L),C50(L)
620   FORMAT(1H ,*CONFIDENCE LEVELS FOR SYSTEM*,I3,//,1H ,9H99% LEVEL,
     CF8.6,/,1H ,9H95% LEVEL,F8.6,/,1H ,9H90% LEVEL,F8.6,/,
     C1H ,9H80% LEVEL,F8.6,/,1H ,9H70% LEVEL,F8.6,/,1H ,9H60% LEVEL,
     CF8.6,/,1H ,9H50% LEVEL,F8.6)
625   CONTINUE
240   CONTINUE
      PRINT 700,((K,NCOUNT(K)),K=1,5)
700   FORMAT(1H ,"COMPONENT",I3,I5,"COMPONENT RELIABILITIES GE 1")
      STOP
      END
```

```
      FUNCTION RANOR(X)
      DIMENSION X(20)
      J=0
    1 SUM=0.
      DO 10 I=1,12
      X(I)=RANF(X)
      SUM=SUM+X(I)
   10 CONTINUE
      Y=SUM-6.0
      RANOR=Y
      RETURN
      END
```

```
      SUBROUTINE RAWEI(EK,TH,XC,T,N,K)
      DIMENSION T(500)
      DO 20 I=1,N
      Y=RANF(X)
      Z=-(ALOG(1.0-Y))
      T(I)=(TH*Z**(1.0/EK))+XC
   20 CONTINUE
   22 ML=1
      MHT=N-1
      ISW=1
      DO 50 I=ML,MHT
      IF(T(I)-T(I+1))50,50,25
   25 GO TO (30,40),ISW
   30 ISW=2
   40 TEMP=T(I)
      T(I)=T(I+1)
      T(I+1)=TEMP
   50 CONTINUE
      GO TO(55,22),ISW
   55 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PARES(N,M,C,THETA,EK,MR,SS1,SEK,T,NY)
CIEWPM HARTER ITERATIVE ESTIMATION OF WEIBULL PARAMETERS MOD 5 APR 1966
C     INPUT
C     N=SAMPLE SIZE (BEFORE CENSORING),N=100 OR LESS AS DIMENSIONED
C     SS1=0  IF SCALE PARAMETER THETA IS KNOWN
C     SS1=1  IF SCALE PARAMETER THETA IS TO BE ESTIMATED
C     SS2=0  IF SHAPE PARAMETER K IS KNOWN
C     SS2=1  IF SHAPE PARAMETER K IS TO BE ESTIMATED
C     SS3=0  IF LOCATION PARAMETER C IS KNOWN
C     SS3=1  IF LOCATION PARAMETER C IS TO BE ESTIMATED
C     T(I)=I-TH ORDER STATISTIC OF SAMPLE (I=1,N)
C       (SUBSTITUTE BLANK CARDS FOR UNKNOWN CENSORED OBSERVATIONS)
C     M=NUMBER OF OBSERVATIONS REMAINING AFTER CENSORING N-M FROM ABOVE
C     C(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF C
C     THETA(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF THETA
C     EK(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF K
C     MR=NUMBER OF OBSERVATIONS CENSORED FROM BELOW,NORMALLY 0 INITIALLY
C     OUTPUT
C     N,SS1,SS2,SS3,M,C(1),THETA(1),EK(1),MR--SAME AS FOR INPUT
C     C(J)=ESTIMATE AFTER J-1 ITERATIONS (OR KNOWN VALUE) OF C
C     THETA(J)=ESTIMATE AFTER J-1 ITERATIONS (OR KNOWN VALUE) OF THETA
C     EK(J)=ESTIMATE AFTER J-1 ITERATIONS (OR KNOWN VALUE) OF K
C       (MAXIMUM VALUE OF J AS PRESENTLY DIMENSIONED IS 550)
C     EL=NATURAL LOGARITHM OF LIKELIHOOD FOR C(J),THETA(J),EK(J)
C     REFERENCE
C     HARTER, H. LEON AND MOORE, ALBERT H., MAXIMUM-LIKELIHOOD ESTIMA-
C       TION OF THE PARAMETERS OF GAMMA AND WEIBULL POPULATIONS FROM
C       COMPLETE AND FROM CENSORED SAMPLES, TECHNOMETRICS, 7 (1965),
C       639-643.  ERRATA, 9 (1967), 195
      DIMENSION T(500),C(550),THETA(550),EK(550),X(56),Y(55)
      SS1=1.
      SS2=1.
      SS3=0.
      IF(N)66,66,104
  104 EN=N
      IF(M)66,66,110
  110 EM=M
   31 ELNM=0.
      EMR=MR
      MRP=MR+1
   33 NM=N-M+1
      DO 34 I=NM,N
      EI=I
   34 ELNM=ELNM+ALOG(EI)
      IF (MR) 66,35,74
   74 DO 75 I=1,MR
      EI=I
   75 ELNM=ELNM-ALOG(EI)
   35 DO 30 J=1,550
      IF (J-1) 66,25,37
   37 JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
    6 SK=SK+(T(I)-C(JJ))**EK(JJ)
      IF (SS1) 7,7,8
    7 THETA(J)=THETA(JJ)
```

```
      GO TO 9
8     IF (MR) 66,19,20
19    THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)**(1./EK(JJ))
      GO TO 9
20    X(1)=THETA(JJ)
      LS=0
      DO 21 L=1,55
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
      Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)+EK(JJ)*(EN-
     1EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.)-EMR*EK(JJ)*ZRK*EXP(-ZRK
     2)/(X(L)*(1.-EXP(-ZRK)))
      IF (Y(L)) 53,73,54
53    LS=LS-1
      IF (LS+L) 58,55,58
54    LS=LS+1
      IF (LS-L) 58,56,58
55    X(LP)=.5*X(L)
      GO TO 61
56    X(LP)=1.5*X(L)
      GO TO 61
58    IF (Y(L)*Y(LL)) 60,73,59
59    LL=LL-1
      GO TO 58
60    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
61    IF (ABS(X(LP)-X(L))-1.E-4) 73,73,21
21    CONTINUE
73    THETA(J)=X(LP)
9     EK(J)=EK(JJ)
10    IF (SS2) 12,12,11
11    DO 17 I=MRP,M
17    SL=SL+ALOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,55
      SLK=0.
      DO 18 I=MRP,M
18    SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))**X(L)
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)**X(L)+(EM-
     1EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)-C(JJ))**X(L)/THETA(J)
     2**X(L)+EMR*ZRK*(ALOG(ZRK)/X(L))*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF (Y(L)) 43,52,44
43    LS=LS-1
      IF (LS+L) 47,45,47
44    LS=LS+1
      IF (LS-L) 47,46,47
45    X(LP)=.5*X(L)
      GO TO 50
46    X(LP)=1.5*X(L)
      GO TO 50
47    IF (Y(L)*Y(LL)) 49,52,48
```

```
48    LL=LL-1
      GO TO 47
49    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
50    IF (ABS(X(LP)-X(L))-1.E-4) 52,52,51
51    CONTINUE
52    EK(J)=X(LP)
12    C(J)=C(JJ)
62    IF (SS3) 25,25,14
14    IF (1.-EK(J)) 16,78,78
78    IF (SS1+SS2) 57,57,16
16    X(1)=C(J)
      LS=0
      DO 23 L=1,55
      SK1=0.
      SR=0.
      DO 15 I=MRP,M
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
15    SR=SR+1./(T(I)-X(L))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))**(EK(J)-1.))
     1/THETA(J)**EK(J)-EMR*EK(J)*ZRK*EXP(-ZRK)/((T(MRP)-X(L))*(1.-EXP
     2(-ZRK)))
      IF (Y(L)) 39,24,40
39    LS=LS-1
      IF (LS+L) 70,41,70
40    LS=LS+1
      IF (LS-L) 70,42,70
41    X(LP)=.5*X(L)
      GO TO 22
42    X(LP)=.5*X(L)+.5*T(1)
      GO TO 22
70    IF (Y(L)*Y(LL)) 72,24,71
71    LL=LL-1
      GO TO 70
72    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
22    IF (ABS(X(LP)-X(L))-1.E-4) 24,24,23
23    CONTINUE
24    C(J)=X(LP)
      GO TO 25
57    C(J)=T(1)
25    IF (MR) 66,38,69
38    DO 63 I=1,M
      IF (C(J)+1.E-4-T(I)) 68,67,67
67    MR=MR+1
63    C(1)=T(1)
68    IF (MR) 66,69,31
69    SK=0.
      SL=0.
      DO 36 I=MRP,M
      SK=SK+(T(I)-C(J))**EK(J)
36    SL=SL+ALOG(T(I)-C(J))
      ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
      EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+(EK(J)-1.)*SL-
     1(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA(J)**EK(J))+EMR*ALOG(1.-EXP
```

```
     2(-Z9K))
  150 IF(J-3)30,27,27
27    IF (ABS(C(J)-C(JJ))-1.E-4) 28,28,30
28    IF (ABS(THETA(J)-THETA(JJ))-1.E-4) 29,29,30
   29 IF(ABS(EK(J)-EK(JJ))-1.E-4)126,126,30
30    CONTINUE
  126 PTH=THETA(J)
      PEK=EK(J)
      GO TO 140
   66 PRINT 135
  135 FORMAT(1H ,23HALL SAMPLES CENSORED,/)
      PEK=0.
      PTH=0.
  140 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE VACOV(Q1,Q2,EK,SNUM,TH,S11,S12,S22)
      K=0
      J=0
      IF(Q1.EQ.1.)GO TO 1
      IF(Q2.EQ.1.)GO TO 1
      GO TO 3
    1 PRINT 2
    2 FORMAT(1H ,14HALL N CENSORED)
      GO TO 27
    3 CONTINUE
      IF (Q1.EQ.0.)GO TO 4
      ZR=(-ALOG(1.00-Q1))**(1.00/EK)
      FLZR=EK*ZR**(EK-1.00)* EXP(-ZR**EK)
      GO TO 5
    4 ZR=0.0
      K=1
      Q1=1.E-28
    5 IF(Q2.EQ.0.)GO TO 7
      ZM=(-ALOG((Q2)))**(1.00/EK)
      GO TO 8
    7 ZM=40.0
      J=1
      Q2=1.E-28
    8 P=1.00-Q1-Q2
      IF(J.EQ.1)GO TO 14
      V11=-EK*P+EK*(EK+1.0)*GAMI(ZM**EK,2.0)+EK*(EK+1.0)
     C    *Q2*ZM**EK
      V22=P/EK**2+D2GAMI(ZM**EK,2.00)/EK**2+Q2*ZM**EK*ALOG(ZM)**2
      V12=P-DGAMI(ZM**EK,2.0)-GAMI(ZM**EK,2.0)
     C    -Q2*ZM**EK*(1.00+EK*ALOG(ZM))
      GO TO 17
   14 V11=-EK*P+EK*(EK+1.0)
      V22=P/EK**2+D2GAM(2.00)/EK**2
      V12=P-DGAM(2.0)-1.0
   17 IF(K.EQ.1)GO TO 24
      V11=V11+(-EK*(EK+1.0)*GAMI(ZR**EK,2.0)+Q1*ZR
     C    *FLZR* (EK*ZR**EK-(EK+1.00)*Q1)/Q1**2)
      V22=V22-D2GAMI(ZR**EK,2.00)/EK**2+Q1*ZR*FLZR*ALOG(ZR)**2*
     C     (ZR**EK-Q1)/(EK*Q1**2)
      V12=V12+DGAMI(ZR**EK,2.0)+GAMI(ZR**EK,2.0)-Q1
     C    *ZR*FLZR*(EK*ZR**EK*ALOG(ZR)-(EK*ALOG(ZR)+1.00)*Q1)/(EK*Q1**2)
   24 D=V11*V22-V12**2
      S11=V22/D
      S12=-V12/D
      S21=S12
      S22=V11/D
      S11=S11*(TH**2)/SNUM
      S12=S12*TH/SNUM
      S22=S22*(1./SNUM)
   27 CONTINUE
      RETURN
      END
```

```
      FUNCTION GAM(Y)
      Z=Y
      G=0.00
1     IF (Z-9.00) 2,2,3
2     G=G-ALOG(Z)
      Z=Z+1.00
      GO TO 1
3     GAM=G+(Z-.500)*ALOG(Z)-Z+.500*ALOG(2.00*3.14159265358979)+1.0/
     C(12.00*Z)-1.00/(360.00*Z**3)+1.00/(1260.00*Z**5)-1.00/(1680.00*Z**
     C7)+1.00/(1188.00*Z**9)-691.00/(360360.00*Z**11)+1.00/(156.00*Z**13
     C)
      GAM= EXP(GAM)
      RETURN
      END
```

```
      FUNCTION DGAM(Y)
      Z=Y
      DG=0.00
1     IF (Z-9.00) 2,2,3
2     DG=DG-1.00/Z
      Z=Z+1.00
      GO TO 1
3     DGAM=DG+(Z-.500)/Z+ALOG(Z)-1.00-1.00/(12.00*Z**2)+1.00/(120.00*Z**
     C     4)-1.00/(252.00*Z**6)+1.00/(240.00*Z**8)-1.00/(132.00*Z**10)
     C     +691.00/(32760.00*Z**12)-1.00/(12.00*Z**14)
      DGAM=DGAM*GAM(Y)
      RETURN
      END
```

```
      FUNCTION D2GAM(Y)
      Z=Y
      D2G=0.00
1     IF (Z-9.00) 2,2,3
2     D2G=D2G+1.00/Z**2
      Z=Z+1.00
      GO TO 1
3     D2GAM=D2G+1.00/Z+1.00/(2.00*Z**2)+1.00/(6.00*Z**3)-1.00/(
     C      5)+1.00/(42.00*Z**7)-1.00/(30.00*Z**9)+5.00/(66.00*Z
     C     -691.00/(2730.00*Z**13)+7.00/(6.00*Z**15)
      D2GAM=(D2GAM*GAM(Y)**2+DGAM(Y)**2)/GAM(Y)
      RETURN
      END
```

```
      FUNCTION GAMI(W,Z)
      DIMENSION U(50)
      U(1)=W**Z/Z
      SU=U(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=(-U(LL)/ELL)*W*(Z+ELL-1.00)/(Z+ELL)
1     SU=SU+U(L)
      GAMI=SU
      RETURN
      END
```

```
      FUNCTION DGAMI(W,Z)
      DIMENSION U(50),V(50)
      U(1)=W**Z*ALOG(W)/Z
      V(1)=W**Z/Z**2
      SU=U(1)-V(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=(-U(LL)*W/ELL)*(Z+ELL-1.00)/(Z+ELL)
      V(L)=-V(LL)*W*(Z+ELL-1.00)**2/((Z+ELL)**2*ELL)
1     SU=SU+U(L)-V(L)
      DGAMI=SU
      RETURN
      END
```

```
      FUNCTION D2GAMI(W,Z)
      DIMENSION U(50),V(50),X(50)
      U(1)=W**Z*ALOG(W)**2/Z
      V(1)=2.00*W**Z*ALOG(W)/Z**2
      X(1)=2.00*W**Z/Z**3
      SU=U(1)-V(1)+X(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=-U(LL)*W*(Z+ELL-1.00)/((Z+ELL)*ELL)
      V(L)=-V(LL)*W*(Z+ELL-1.00)**2/((Z+ELL)**2*ELL)
      X(L)=-X(LL)*W*(Z+ELL-1.00)**3/((Z+ELL)**3*ELL)
1     SU=SU+U(L)-V(L)+X(L)
      D2GAMI=SU
      RETURN
      END
```

C. Univariate Technique: Notes and Computer Program Listings

The values of the seeds, T and DSEED, used to generate random numbers in the univariate technique are presented in Tables LVII and LVIII. The same seed values were used for the univariate-normal case, the univariate-beta case, and when applying Gatliffe's technique to the univariate method. As stated in Section III, the same seed values were used so that only the underlying distribution of component reliability estimates varied.

The variable T is real in type. The variable DSEED is double precision in type and has the format

XXXX.ODO

where XXXX is one of the integer values presented in Table LVIII. DSEED and T are used in the generation of Weibull random deviates and normal random deviates in the univariate-normal technique and in the generation of Weibull random deviates and beta random deviates in the univariate-beta technique. The code for each technique is also presented in this appendix.

The computer programs for the univariate-normal and univariate-beta cases differ only in one section of the main program WEIB. All subroutines for both cases are exactly the same. Therefore, a listing of the entire computer

TABLE LVII

Values of Variable T

Number Simulated Points	Component Sample Size 10	15	20	50	100
100	6867	108	3049	384	2344
200	3730	6510	6063	7602	5202
	7972	517	7905	9432	3260
	3860	8931	6779	9168	7978
600	1763	5397	7486	1085	8492
	5675	2501	7238	945	7257
	283	3170	7628	7272	3355
	5269	5173	1169	1644	9747
	6403	7840	4341	5716	979
	9776	6306	7834	4311	8221
1000	9474	3971	1460	3333	9261
	7860	6048	9334	5972	6755
	4165	4673	1204	4086	3268
	7176	9250	7116	6098	4207
	2883	882	4311	2741	2020
	6904	7242	1157	9490	1043

program for the univariate-normal case is presented first. Following this is the listing of the main program WEIB for the univariate-beta case with the differing lines of code boxed.

TABLE LVIII

Values of Variable DSEED

Number Simulated Points	Component Sample Size				
	10	15	20	50	100
100	2361	698	1589	3767	4576
200	5955	382	6711	2158	9463
	6630	3751	9314	5353	4723
	4334	928	9783	5313	5105
600	7966	4678	3327	1810	9342
	9660	5234	8405	7590	4618
	6404	5648	6083	3047	4767
	8712	5744	7522	4733	199
	6771	162	6466	8113	5352
	8101	107	8834	5384	3958
100	4992	5086	5185	1301	3868
	8902	9260	5122	2116	9596
	4112	6666	7778	9289	6567
	1325	6555	25	2252	2434
	6531	515	3757	7600	7370
	7413	3172	6935	9563	4330

These programs were run on a CDC 6600 machine. They occupy 140,000 words of core and vary in execution time. The execution times for each computer run are listed in Table LIX by number of simulated points, type of univariate technique, and component sample size. For any given

TABLE LIX

Univariate Technique: Execution Times (Seconds)

Number Simulated Reliability Points	Component Sample Size 10	15	20	50	100
600 100: simulations per run					
Normal	383	456	519	919	1578
Beta	335	389	452	796	1684
200 200: simulations per run					
Normal	146	168	188	306	491
Beta	135	151	171	283	470
100 600: simulations per run					
Normal	126	137	148	209	306
Beta	108	114	123	185	279
100 1000: simulations per run					
Normal	179	187	200	257	359
Beta	145	149	159	217	314

component sample size, all 600 Monte Carlo simulations were made in one computer run for the case of 100 simulated points. Execution time ranged from 383 seconds for a

component sample size of 10 to 1578 seconds for the component sample size of 100. Because of the large amount of execution time required, three computer runs of 200 Monte Carlo simulations each were performed for the cases of 200 simulated reliability points and six computer runs of 100 Monte Carlo simulations each were performed for the cases of 600 and 1000 system reliability points in order to obtain the 600 Monte Carlo simulation run results. Thus, the tabled execution times are not the times required for the 600 Monte Carlo simulations except in the case of 100 simulated points. This also explains the varying number of seed values listed in Tables LVII and LVIII. For example, in the case of 600 simulated reliability points for a component sample size of 10, six computer runs of 100 Monte Carlo simulations were made for the univariate-normal technique. Each of the six computer runs required new values for T and DSEED (thus six for each variable are listed) and 126 seconds of execution time.

```
      PROGRAM WEIB(INPUT,OUTPUT)
C
C  ********************************************************************
C  *                                                                  *
C  * FOR EACH COMPONENT OF A COMPLEX SYSTEM, THIS PROGRAM             *
C  * GENERATES A SAMPLE OF SIZE NSAM FROM THE WEIBULL DISTRIBUTION    *
C  * USING THE TRUE PARAMETERS TK (SHAPE), TTHETA (SCALE), AND TC     *
C  * (LOCATION).  FROM THIS SAMPLE, THE MAXIMUM LIKELIHOOD            *
C  * ESTIMATORS (MLES) FOR K AND THETA ARE DERIVED USING HARTER &      *
C  * MOORES ITERATIVE SCHEME AND THE SIMULTANEOUS EQUATION SOLVER      *
C  * ZSYSTM.  THE MLES ARE COMBINED TO YIELD RCHAT, THE MLE FOR THE   *
C  * COMPONENT RELIABILITY.  GIVEN THE RELIABILITY AND THE SAMPLE     *
C  * SIZE NSAM, RCHAT IS ASYMPTOTICALLY NORMALLY DISTRIBUTED WITH A   *
C  * SPECIFIED VARIANCE.  THEREFORE WE CAN SAMPLE FROM THE NORMAL     *
C  * DISTRIBUTION TO OBTAIN A VECTOR OF SAMPLE COMPONENT              *
C  *RELIABILITIES.  THIS PROCESS IS REPEATED FOR EACH COMPONENT       *
C  * AND THEN THE RELIABILITIES ARE COMBINED FOR 4 DIFFERENT TYPES    *
C  * OF SYSTEMS TO YIELD 4 VECTORS OF SAMPLE SYSTEM RELIABILITIES.    *
C  * THESE VECTORS ARE ORDERED AND THEN THE 95, 90, AND 80 PERCENT    *
C  * LOWER CONFIDENCE LIMITS ARE PICKED.  THIS ESTABLISHES THE 95,    *
C  * 90, AND 80 PERCENT CONFIDENCE INTERVALS FOR EACH SYSTEM AND IT   *
C  * IS NOTED WHETHER EACH OF THESE INTERVALS CONTAINS THE TRUE       *
C  * SYSTEM RELIABILITY.                                              *
C  * THE ABOVE PROCESS IS REPEATED FOR NOLMC MONTE CARLO RUNS, WITH   *
C  * COUNTERS FOR EACH SYSTEM TO TRACK THE NUMBER OF TIMES THAT THE   *
C  * CONFIDENCE INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.        *
C  *                                                                  *
C  ********************************************************************
C
      DIMENSION CC(550),THETA(550),EK(550)
      DIMENSION  ARCHAT(2000,5), ATRS(4), BI(12,15), C(4,4), DEV
     1 (200), PARAM(3,5), R(500,5),RC(20),5),RCHAT
     2  (5), RLBS(12), RS(200), SIG(12,15), SIGSQ
     3 (12,15), SMSZ(15), TEMP(200), TRC(5), TRS(1),
     4  WK(200)
      DOUBLE PRECISION DSEED
      REAL KHAT
      DATA RLBS / .5, .55, .6, .65, .7, .75, .8, .85, .9, .925, .95,
     1  .98 /
C
      DATA SMSZ / 8., 9., 10., 11., 12., 13., 14., 15., 20., 25.,
     1  30., 40., 50., 75., 100. /
C
C  INITIALIZE VARIABLES
      PI      =4. * ATAN(1.)
      READ*,DSEED,T
      CALL RANSET (T)
      TIME    =100.
      NRLBS   =12
      NSMSZ   =15
      NSMSZ1  =NSMSZ - 1
      NRLBS1  =NRLBS - 1
      ISD     =12
      NTRUNC  =0
      LTRUNC=0
      NS1C99  =0
      NS1C95  =0
```

```
      NS1C90     =0
      NS1C80     =0
      NS1C70     =0
      NS1C60     =0
      NS1C50     =0
      NS2C99     =0
      NS2C95     =0
      NS2C90     =0
      NS2C80     =0
      NS2C70     =0
      NS2C60     =0
      NS2C50     =0
      NS3C99     =0
      NS3C95     =0
      NS3C90     =0
      NS3C80     =0
      NS3C70     =0
      NS3C60     =0
      NS3C50     =0
      NS4C99=0
      NS4C95     =0
      NS4C90     =0
      NS4C80     =0
      NS4C70     =0
      NS4C60     =0
      NS4C50     =0
C
      PRINT  409
C  READ THE SAMPLE SIZE AND NUMBER OF MONTE CARLO RUNS
      READ*,NMCT,IDIMT,NPTS
      READ*,M1,M2,M3,M4,M5,M6,M7
      READ *, NSAM, NOLMC
      PRINT  419, NSAM, NOLMC
      RNSAM     =NSAM
      M=NSAM
      MR=0
C  READ THE TRUE COMPONENT PARAMETERS
      READ *, ((PARAM(I, J), I=1, 3), J=1, 5)
      PRINT  429,  ((J, (PARAM(I, J), I=1, 3), CMPREL(TIME,PARAM(1, J),
     1  PARAM(2, J), PARAM(3, J))), J=1, 5)
      READ*,((SIG(I,J),J=1,15), I=1,12)
      READ*,((BI(I,J),J=1,15),I=1,12)
      PRINT  439, ((SIG(I, J), J=1, 15), I=1,12)
      PRINT  439,  ((BI(I, J), J=1, 15), I=1,12)
C
C  ***********************************************************
C  ***********************************************************
C  THIS OUTSIDE LOOP FROM HERE TO STATEMENT 300 COMPLETES
C  NOLMC MONTE CARLO RUNS OF THE SIMULATION.
C
        DO 300 NCOUNT = 1, NOLMC
C
C  ***********************************************************
C
C  FOR EACH OF 5 COMPONENTS, THIS LOOP GENERATES THE MLE OF RCHAT
C  FROM THE MLES OF K AND THETA.  IT DETERMINES THE VARIANCE OF
C RCHAT AND THEN SAMPLES FROM THE NORMAL DISTRIBUTION FOR NDEV
```

```
C  SAMPLE RELIABILITIES OF EACH COMPONENT.
C
      DO 290 J =1, 5
      TK      = PARAM(1, J)
      TTHETA  = PARAM(2, J)
      TC      = PARAM(3, J)
C  PASS INFORMATION ON COMPONENT NUMBER AND
C  SAMPLE SIZE THRU ARRAY R
      R(205, 1) = J
      R(204, J) = NSAM
C  DETERMINE THE TRUE COMPONENT RELIABILITY
      TRL(J)    =CMPREL(TIME, TK, TTHETA, TC)
C
C  GATHER NSAM FAILURE TIMES FROM THE
C  WEIBULL DISTRIBUTION WITH TRUE PARAMETERS
C  TK,TTHETA, AND TC
C
      CALL GGWIB (DSEED, TK, NSAM, TEMP)
        DO 20 I   =1, NSAM
        P(I, J)   =TTHETA * TEMP(I) + TC
   20   CONTINUE
C  DETERMINE THE LIKELIHOOD OF DRAWING THIS SAMPLE
C  USING THE TRUE K, THETA, AND C
C
C  DETERMINE THE MLE OF THETA AND K
C  BY HARTER & MOORES ITERATIVE SCHEME
C
      CC(1)=TC
      THETA(1)=TTHETA
      EK(1)=TK
      CALL PARES(NSAM,M,CC,THETA,EK,MR,THAT,KHAT,R(1,J))
C
C  DETERMINE RCHAT, THE MLE OF THE COMPONENT RELIABILITY, USING
C  THE TRUE C AND THE MLES OF K AND THETA
      RCHAT(J) = CMPREL(TIME, KHAT, THAT, TC)
C
C  GIVEN THE MLE OF THE COMPONENT RELIABILITY AND THE SAMPLE SIZE,
C  ENTER THE 2-DIMENSIONAL ARRAY BI AND FIND THE BIAS OF THE
C  ESTIMATOR
        DO 50 I   =1, NSMSZ1
        LSMSZ     =I
        IF (RNSAM .LE. SMSZ(I+1)) GO TO 60
   50   CONTINUE
      GO TO 90
   60 IF (RCHAT(J) .LT. .5) GO TO 100
        DO 70 I   =1, NRLBS1
        LRLBS     =I
        IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 80
   70   CONTINUE
      GO TO 90
   80 CALL ISCICU (BI, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ,
     1    C, WK, IER)
      CALL ISCEVU (RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ, C,
     1    RCHAT(J), RNSAM, BIAS, IER)
      IF (LRLBS .LT. NRLBS) GO TO 110
   90 BIAS    =0.
```

```
            GO TO 110
  100     IF (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ = LSMSZ + 1
          BIAS    = BI(1, LSMSZ)
  110       RCHAT(J)  =RCHAT(J) - BIAS
            ARCHAT(NCOUNT, J) = RCHAT(J)
C
C  GIVEN THE UNBIASED ESTIMATOR OF THE COMPONENT RELIBILITY,
C  ENTER THE 2-DIMENSIONAL ARRAY SIG AND FIND THE STANDARD
C  DEVIATION OF THE UNBIASED ESTIMATOR
            Z         =RCHAT(J)
            RCRLB     =SQRT(Z**2*(ALOG(Z))**2*(1.109-.514*
     1        ALOG(-ALOG(Z)) + .608*(ALOG(-ALOG(Z)))**2)/RNSAM)
            DO 120 I  =1, NSMSZ1
            LSMSZ     =I
            IF (RNSAM .LE. SMSZ(I+1)) GO TO 130
  120       CONTINUE
            GO TO 160
  130       IF (RCHAT(J) .LT. .5) GO TO 170
            DO 140 I  =1, NRLBS1
            LRLBS     =I
            IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 150
  140       CONTINUE
            GO TO 160
  150       CALL IBCICU (SIG, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS,
     1        LSMSZ, C, WK, IER)
            CALL IBCEVU (RLBS, NRLBS, SMSZ,NSMSZ, LRLBS, LSMSZ, C,
     1        RCHAT(J), RNSAM, SIGMA, IER)
            IF (SIGMA .GE. RCRLB) GO TO 180
  160       SIGMA     =RCRLB
            IER       =0
            GO TO 180
  170       IF (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ=LSMSZ + 1
            SIGMA     =SIG(1, LSMSZ)
            IF (SIGMA .GE. RCRLB) GO TO 180
            SIGMA     =RCRLB
  180       CONTINUE
C
C  FORM A VECTOR OF NDEV SAMPLE RELIABILITIES WITH MEAN RCHAT
C  AND A STANDARD DEVIATION OF SIGMA
          NDEV=NMCT
            CALL GGNML (DSEED, NDEV, DEV)
          DO 190 I=1,NDEV
              RC(I, J)  =RCHAT(J) + DEV(I) * SIGMA
          IF(RC(I,J).LT.0.0) LTRUNC=LTRUNC+1
          IF(RC(I,J).LT.0.0) RC(I,J)=0.0
C  TRUNCATE THE NORMAL DISTRIBUTION IF COMPONENT RELIABILITY
C  IS GREATER THAN 1
            IF (RC(I,J) .LE. 1.) GO TO 190
            RC(I,J)  =1.
            NTRUNC   =NTRUNC + 1
  190 CONTINUE
  200     CONTINUE
C  ********************************************************************
C
C  FOR EACH OF THE 4 SYSTEMS, THE DIFFERENT COMPONENTS ARE COMBINED
C  TO YIELD NDEV SAMPLES OF THE SYSTEM RELIABILITY.  THESE SAMPLES
C  ARE SEQUENCED IN ASCENDING ORDER AND THEN COUNTERS KEEP TRACK
```

```
C  OF WHEN THE 99, 95, 90, 80, 70, 60, AND 50 PERCENT CONFIDENCE
C  INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.
C
C  SYSTEM 1
           NMC      =1
           IDIM     =1
           CALL REL1 (NMC, TRC, TRS, IDIM)
           ATRS(1)  =TRS(1)
        NMC=NMCT
        IDIM=IDIMT
           CALL REL1 (NMC, RC, RS, IDIM)
           CALL VSRTA (RS, IDIM)
        IF(RS(M1).LE.TRS(1)) NS1C99=NS1C99+1
        IF(RS(M2).LE.TRS(1)) NS1C95=NS1C95+1
        IF(RS(M3).LE.TRS(1)) NS1C90=NS1C90+1
        IF(RS(M4).LE.TRS(1)) NS1C80=NS1C80+1
        IF(RS(M5).LE.TRS(1)) NS1C70=NS1C70+1
        IF(RS(M6).LE.TRS(1)) NS1C60=NS1C60+1
        IF(RS( M7).LE.TRS(1)) NS1C50=NS1C50+1
C
C  SYSTEM 2
           NMC      =1
           IDIM     =1
           CALL REL2 (NMC, TRC, TRS, IDIM)
           ATRS(2)  =TRS(1)
        NMC=NMCT
        IDIM=IDIMT
           CALL REL2 (NMC, RC, RS, IDIM)
           CALL VSRTA (RS, IDIM)
        IF(RS(M1).LE.TRS(1)) NS2C99=NS2C99+1
        IF(RS(M2).LE.TRS(1)) NS2C95=NS2C95+1
        IF(RS(M3).LE.TRS(1)) NS2C90=NS2C90+1
        IF(RS(M4).LE.TRS(1)) NS2C80=NS2C80+1
        IF(RS(M5).LE.TRS(1)) NS2C70=NS2C70+1
        IF(RS(M6).LE.TRS(1)) NS2C60=NS2C60+1
        IF(RS( M7).LE.TRS(1)) NS2C50=NS2C50+1
C
C  SYSTEM 3
           NMC      = 1
           IDIM     = 1
           CALL REL3 (NMC, TRC, TRS, IDIM)
           ATRS(3)  = TRS(1)
        NMC=NMCT
        IDIM=IDIMT
           CALL REL3 (NMC, RC, RS, IDIM)
           CALL VSRTA (RS, IDIM)
        IF(RS(M1).LE.TRS(1)) NS3C99=NS3C99+1
        IF(RS(M2).LE.TRS(1)) NS3C95=NS3C95+1
        IF(RS(M3).LE.TRS(1)) NS3C90=NS3C90+1
        IF(RS(M4).LE.TRS(1)) NS3C80=NS3C80+1
        IF(RS(M5).LE.TRS(1)) NS3C70=NS3C70+1
        IF(RS(M6).LE.TRS(1)) NS3C60=NS3C60+1
        IF(RS( M7).LE.TRS(1)) NS3C50=NS3C50+1
C
C  SYSTEM 4
           NMC      = 1
           IDIM     = 1
```

```
            CALL REL4 (NMC, TRC, TRS, IDIM)
            ATRS(4)  = TRS(1)
          NMC=NMCT
          IDIM=IDIMT
            CALL REL4 (NMC, RC, RS, IDIM)
            CALL VSRTA (RS, IDIM)
          IF(RS(M1).LE.TRS(1)) NS4C99=NS4C99+1
          IF(RS(M2).LE.TRS(1)) NS4C95=NS4C95+1
          IF(RS(M3).LE.TRS(1)) NS4C90=NS4C90+1
          IF(RS(M4).LE.TRS(1)) NS4C80=NS4C80+1
          IF(RS(M5).LE.TRS(1)) NS4C70=NS4C70+1
          IF(RS(M6).LE.TRS(1)) NS4C60=NS4C60+1
          IF(RS( M7).LE.TRS(1)) NS4C50=NS4C50+1
  300     CONTINUE
C
C  ***********************************************************
C  ***********************************************************
C
          NPT=5*NPTS*NOLMC
          PRINT 350, NTRUNC
  350   FORMAT(1H ,11HTHERE WERE ,I7,18H LOWER TRUNCATIONS)
          PRINT 469,NTRUNC,NPT
          RNOLMC     = NOLMC
C  FOR SYSTEM 1, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
          PS1C99     = NS1C99 / RNOLMC
          PS1C95     = NS1C95 / RNOLMC
          PS1C90     = NS1C90 / RNOLMC
          PS1C80     = NS1C80 / RNOLMC
          PS1C70     = NS1C70 / RNOLMC
          PS1C60     = NS1C60 / RNOLMC
          PS1C50     = NS1C50 / RNOLMC
C  FOR SYSTEM 2, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
          PS2C99     = NS2C99 / RNOLMC
          PS2C95     = NS2C95 / RNOLMC
          PS2C90     = NS2C90 / RNOLMC
          PS2C80     = NS2C80 / RNOLMC
          PS2C70     = NS2C70 / RNOLMC
          PS2C60     = NS2C60 / RNOLMC
          PS2C50     = NS2C50 / RNOLMC
C  FOR SYSTEM 3, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
          PS3C99     = NS3C99 / RNOLMC
          PS3C95     = NS3C95 / RNOLMC
          PS3C90     = NS3C90 / RNOLMC
          PS3C80     = NS3C80 / RNOLMC
          PS3C70     = NS3C70 / RNOLMC
          PS3C60     = NS3C60 / RNOLMC
          PS3C50     = NS3C50 / RNOLMC
C  FOR SYSTEM 4, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
          PS4C99     = NS4C99 / RNOLMC
          PS4C95     = NS4C95 / RNOLMC
          PS4C90     = NS4C90 / RNOLMC
          PS4C80     = NS4C80 / RNOLMC
          PS4C70     = NS4C70 / RNOLMC
```

```
      PS4C60    = NS4C60 / RNDLMC
      PS4C50    = NS4C50 / RNDLMC
      PRINT  479
      PRINT 489,ATRS(1),PS1C99,PS1C95,PS1C90,PS1C80,PS1C70,PS1C60,PS1C50
       PRINT 499
      PRINT 489,ATRS(2),PS2C99,PS2C95,PS2C90,PS2C80,PS2C70,PS2C60,PS2C50
       PRINT  509
      PRINT 489,ATRS(3),PS3C99,PS3C95,PS3C90,PS3C80,PS3C70,PS3C60,PS3C50
       PRINT  519
      PRINT 489,ATRS(4),PS4C99,PS4C95,PS4C90,PS4C80,PS4C70,PS4C60,PS4C50
C
C     ********************************************************************
C     ********************************************************************
C
      STOP "FORAWHILE"
C
  409 FORMAT  ( 1H1  )
  419 FORMAT  ( " **********",  /  ,  " *", T62,  "*",  /  ,  " *", T52,  "*",  / C
     1   "**********",  /  ,  " *", T62,  "*",  /  ,  " *", T52,  "*",  / C
     2   ,  " *", T25,  "SAMPLE "  "SIZE = ", I3, T62,  "*",  /  ,  " *",C
     3   T62,  "*",  /  ,  " *", T22,  "MONTE CARLO "  "SIZE = ", I3,  C
     4   T62,  "*",  /  ,  " *", T62,  "*",  /  ,  " *", T62,  "*",  /  ,
     5   " **********",
     6   "*",  /  /  /  /  /  /  /  )
  429 FORMAT  ( 5(1X,  "COMPONENT ", I1,  / , 6X,  "K = ", F4.2,  / ,  C
     1   6X,  "THETA = ", F5.0,  / , 6X,  "C = ", F2.0,  / , 6X,
     2   "RELIABILITY =", F7.5,  /  / ) )
  439 FORMAT  ( 15(1X, F7.5) )
  449 FORMAT  (/," **********",/," MC=",I4,4X,"J =",I2,4X,"NSTART =",I2
     1                          ,/," KHAT=" E13.6, 5X "THAT=" E13.6 /C
     2   " ITERATIONS=" I3, 5X "IER=" I3 /  " **********"  )
  459 FORMAT  (  /  " ********************" /  /  " DID NOT CONVERGE IN"C
     1   " 8 ATTEMPTS  WITH DIFFERENT STARTING KHATS" /  " THEREFORE GO"C
     2   "ING ON TO THE NEXT  COMPONENT" /  /  " ********************" C
     3   )
  469 FORMAT  ( /  /  ,  " THERE WERE", I7,  " HIGH TRUNCATIONS OUT OF"
     1   , I8, " RELIABILITY DEVIATES", / / )
  479 FORMAT  ( /  /  " ***** SYSTEM 1 *****" /  " (3 COMPONENTS IN SERI"C
     1   "ES)"  )
  489 FORMAT  ( /  ,  " TRUE SYSTEM RELIABILITY =", F7.5,  /
     1   " THE 99 PERCENT CONFIDENCE INTERVAL COVERED ",F6.4,
     2   " OF THE RUNS",/,
     3   " THE 95 PERCENT CONFIDENCE INTERVAL COVERED ", F6.4,
     4   " OF THE RUNS",/," THE 90 PERCENT CONFIDENCE INTERVAL COVERED "
     5 , F6.4," OF THE RUNS",/," THE 80 PERCENT CONFIDENCE INTERVAL"
     6   " COVERED ",F6.4," OF THE RUNS",/," THE 70 PERCENT CONFIDENCE "
     7   "INTERVAL COVERED ",F6.4," OF THE RUNS",/," THE 60 PERCENT "
     8   "CONFIDENCE INTERVAL COVERED ",F6.4," OF THE RUNS",/," THE",
     9   " 50 PERCENT CONFIDENCE INTERVAL COVERED ",F6.4," OF THE RUNS",
     1   ///)
  499 FORMAT  (  /  " ***** SYSTEM 2 *****" /  " (1 COMPONENT IN SERIE"C
     1   "S WITH 2 "  "IN PARALLEL)"  )
  509 FORMAT  (  /  " ***** SYSTEM 3 *****" /  " (3 COMPONENTS IN PARA"C
     1   "LLEL)" )
  519 FORMAT  (  /  " ***** SYSTEM 4 *****" /  " (A 5-COMPONENT COMPLE"C
     1   "X NETWORK)"  )
  529 FORMAT  ( 5(1X, F10.5, 5X) )                                      C

C
      END
```

```
      FUNCTION CMPREL (TIME, K, THETA, C)
      REAL       K
      IF(THETA.EQ.TIME) CMPREL=.36787944
      IF(THETA.EQ.TIME) GO TO 9
      IF(THETA.LT.10E9 .AND.THETA .GE.10E-7) GO TO 6
      IF(THETA.GE.10E9) CMPREL=1.0
      IF(THETA.GE.0.0 .AND.THETA.LT.10E-7) CMPREL=0.0
      IF(THETA.LT.0.0) PRINT 5
    5 FORMAT(1H ,17HTHETA IS NEGATIVE)
      IF(THETA.LT.0.0) CMPREL=0.0
      RETURN
    6 ARGUM=-((TIME-C)/THETA)**K
      IF(ARGUM.GT.-20.0 .AND.ARGUM.LT.0.0) GO TO 8
      IF(ARGUM.GE.0.0) CMPREL=1.0
      IF(ARGUM.LE.-20.0) CMPREL=0.0
      RETURN
    8 CMPREL=EXP(ARGUM)
    9 RETURN
      END
```

```
      SUBROUTINE  REL1 (NMC, R, RS, IDIM)
C  REL1 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN SERIES
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   =1, NMC
   10   RS(I)     =R(I, 1) * R(I,2) * R(I,3)
      RETURN
      END
```

```
      SUBROUTINE  REL2 (NMC, R, RS, IDIM)
C  REL2 DETERMINES THE SYSTEM RELIABILITY OF 1 COMPONENT IN
C  SERIES WITH 2 IN PARALLEL
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   =1, NMC
   10   RS(I)     =R(I, 1) * (1. - (1. -R(I, 2)) * (1. -R(I, 3)))
      RETURN
      END
```

```
      SUBROUTINE  REL3 (NMC, R, RS, IDIM)
C  REL3 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN PARALLEL
      DIMENSION   R (IDIM,5)  RS (IDIM)
        DO 10 I   =1, NMC
   10   RS(I)     =1. - (1. -R(I,1)) * (1. -R(I, 2)) * (1. -R(I,
     1    3))
      RETURN
      END
```

```
      SUBROUTINE  REL4 (NMC, R, RS, IDIM)
C  REL4 DETERMINES THE SYSTEM RELIABILITY OF A 5 COMPONENT
C  COMPLEX NETWORK
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   =1, NMC
        RS(I)     =R(I, 1) * (1.-(1.-R(I, 2)) * (1.-R(I,5) * (1.-
     1    (1. - R(I,3)) * (1. - R(I,4)))))
   10   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PARES(N,M,C,THETA,EK,MR,PTH,PEK,T)
C     INPUT
C     N=SAMPLE SIZE (BEFORE CENSORING),N=100 OR LESS AS
C           DIMENSIONED
C     SS1=0 IF SCALE PARAMETER THETA IS KNOWN
C     SS1=1 IF SCALE PARAMETER THETA IS TO BE ESTIMATED
C     SS2=0 IF SHAPE PARAMETER K IS KNOWN
C     SS2=1 IF SHAPE PARAMETER K IS TO BE ESTIMATED
C     SS3=0 IF LOCATION PARAMETER C IS KNOWN
C     SS3=1 IF LOCATION PARAMETER C IS TO BE ESTIMATED
C     T(I)=I-TH ORDER STATISTIC OF SAMPLE (I=1,N)
C     M=NUMBER OF OBSERVATIONS REMAINING AFTER CENSORING N-M
C           FROM ABOVE
C     C(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF C
C     THETA(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF THETA
C     EK(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF K
C     MR=NUMBER OF OBSERVATIONS CENSORED FROM BELOW
C     OUTPUT
C     N,SS1,SS2,SS3,M,C(1),THETA(1),EK(1),MR
C           --SAME AS FOR INPUT
C     C(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF C
C     THETA(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF THETA
C     EK(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF K
C      (MAXIMUM VALUE OF J AS PRESENTLY DIMENSIONED IS 500)
C     EL=NATURAL LOG. OF LIKELIHOOD FOR C(J),THETA(J),EK(J)
      DIMENSION T(500),C(550),THETA(550),EK(550),X(56),Y(55)
      SS1=1.
      SS2=1.
      SS3=0.
      IF(N)66,66,104
  104 EN=N
      IF(M)66,66,110
  110 EM=M
   31 ELNM=0.
      EMR=MR
      MRP=MR+1
   33 NM=N-M+1
      DO 34 I=NM,N
      EI=I
   34 ELNM=ELNM+ALOG(EI)
      IF (MR) 66,35,74
   74 DO 75 I=1,MR
      EI=I
   75 ELNM=ELNM-ALOG(EI)
   35 DO 30 J=1,550
      IF (J-1) 66,25,37
   37 JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
    6 SK=SK+(T(I)-C(JJ))**EK(JJ)
      IF (SS1) 7,7,8
    7 THETA(J)=THETA(JJ)
      GO TO 9
```

```
8     IF (MR) 66,19,20
19    THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)
     C**(1./EK(JJ))
      GO TO 9
20    X(1)=THETA(JJ)
      LS=0
      DO 21 L=1,55
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
      Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)
     C+EK(JJ)*(EN-EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.
     C)-EMR*EK(JJ)*ZRK*EXP(-ZRK)/(X(L)*(1.-EXP(-ZRK)))
      IF (Y(L)) 53,73,54
53    LS=LS-1
      IF (LS+L) 58,55,58
54    LS=LS+1
      IF (LS-L) 58,56,58
55    X(LP)=.5*X(L)
      GO TO 61
56    X(LP)=1.5*X(L)
      GO TO 61
58    IF (Y(L)*Y(LL)) 60,73,59
59    LL=LL-1
      GO TO 58
60    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
61    IF (ABS(X(LP)-X(L))-1.E-4) 73,73,21
21    CONTINUE
73    THETA(J)=X(LP)
9     EK(J)=EK(JJ)
10    IF (SS2) 12,12,11
11    DO 17 I=MRP,M
17    SL=SL+ALOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,55
      SLK=0.
      DO 18 I=MRP,M
18    SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))
     C**X(L)
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)
     C**X(L)+(EN-EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)
     C-C(JJ))**X(L)/THETA(J)**X(L)+EMR*ZRK*(ALOG(ZRK)/X(L))
     C*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF (Y(L)) 43,52,44
43    LS=LS-1
      IF (LS+L) 47,45,47
44    LS=LS+1
      IF (LS-L) 47,46,47
45    X(LP)=.5*X(L)
      GO TO 50
46    X(LP)=1.5*X(L)
```

```
      GO TO 50
47    IF (Y(L)*Y(LL)) 49,52,48
48    LL=LL-1
      GO TO 47
49    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
50    IF (ABS(X(LP)-X(L))-1.E-4) 52,52,51
51    CONTINUE
52    EK(J)=X(LP)
12    C(J)=C(JJ)
62    IF (SS3) 25,25,14
14    IF (1.-EK(J)) 16,78,78
78    IF (SS1+SS2) 57,57,16
16    X(1)=C(J)
      LS=0
      DO 23 L=1,55
      SK1=0.
      SR=0.
      DO 15 I=MRP,M
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
15    SR=SR+1./(T(I)-X(L))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))
     C**(EK(J)-1.))/THETA(J)**EK(J)-EMR*EK(J)*ZRK*EXP(-ZRK)
     C/((T(MRP)-X(L))*(1.-EXP(-ZRK)))
      IF (Y(L)) 39,24,40
39    LS=LS-1
      IF (LS+L) 70,41,70
40    LS=LS+1
      IF (LS-L) 70,42,70
41    X(LP)=.5*X(L)
      GO TO 22
42    X(LP)=.5*X(L)+.5*T(1)
      GO TO 22
70    IF (Y(L)*Y(LL)) 72,24,71
71    LL=LL-1
      GO TO 70
72    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
22    IF (ABS(X(LP)-X(L))-1.E-4) 24,24,23
23    CONTINUE
24    C(J)=X(LP)
      GO TO 25
57    C(J)=T(1)
25    IF (MR) 66,38,69
38    DO 63 I=1,M
      IF (C(J)+1.E-4-T(I)) 68,67,67
67    MR=MR+1
63    C(1)=T(1)
68    IF(MR) 66,69,31
69    SK=0.
      SL=0.
      DO 36 I=MRP,M
      SK=SK+(T(I)-C(J))**EK(J)
36    SL=SL+ALOG(T(I)-C(J))
      ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
```

```
      EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+
     C(EK(J)-1.)*SL-(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA
     C(J)**EK(J))+EMR*ALOG(1.-EXP(-ZRK))
150   IF(J-3) 30,27,27
27    IF (ABS(C(J)-C(JJ))-1.E-4) 28,28,30
28    IF (ABS(THETA(J)-THETA(JJ))-1.E-4) 29,29,30
  29  IF(ABS(EK(J)-EK(JJ))-1.E-4)126,126,30
30    CONTINUE
126   PTH=THETA(J)
      PEK=EK(J)
      GO TO 140
   66 PRINT 135
  135 FORMAT(1H ,20HALL SAMPLES CENSORED,/)
      PEK=0.
      PTH=0.
  140 CONTINUE
      RETURN
      END
```

```
      PROGRAM WEIB(INPUT,OUTPUT)
C
C  ***************************************************************
C  *                                                             *
C  * FOR EACH COMPONENT OF A COMPLEX SYSTEM, THIS PROGRAM        *
C  * GENERATES A SAMPLE OF SIZE NSAM FROM THE WEIBULL DISTRIBUTION*
C  * USING THE TRUE PARAMETERS TK (SHAPE), TTHETA (SCALE), AND TC*
C  * (LOCATION).  FROM THIS SAMPLE, THE MAXIMUM LIKELIHOOD       *
C  * ESTIMATORS (MLES) FOR K AND THETA ARE DERIVED USING HARTER & *
C  * MOORES ITERATIVE SCHEME AND THE SIMULTANEOUS EQUATION SOLVER *
C  * ZSYSTM.  THE MLES ARE COMBINED TO YIELD RCHAT, THE MLE FOR THE*
C  * COMPONENT RELIABILITY.  GIVEN THE RELIABILITY AND THE SAMPLE *
C  * SIZE NSAM, RCHAT IS ASYMPTOTICALLY NORMALLY DISTRIBUTED WITH A*
C  * SPECIFIED VARIANCE.  THEREFORE WE CAN SAMPLE FROM THE NORMAL *
C  * DISTRIBUTION TO OBTAIN A VECTOR OF SAMPLE COMPONENT          *
C  *RELIABILITIES.  THIS PROCESS IS REPEATED FOR EACH COMPONENT   *
C  * AND THEN THE RELIABILITIES ARE COMBINED FOR 4 DIFFERENT TYPES*
C  * OF SYSTEMS TO YIELD 4 VECTORS OF SAMPLE SYSTEM RELIABILITIES.*
C  * THESE VECTORS ARE ORDERED AND THEN THE 95, 90, AND 80 PERCENT*
C  * LOWER CONFIDENCE LIMITS ARE PICKED.  THIS ESTABLISHES THE 95,*
C  * 90, AND 80 PERCENT CONFIDENCE INTERVALS FOR EACH SYSTEM AND IT*
C  * IS NOTED WHETHER EACH OF THESE INTERVALS CONTAINS THE TRUE   *
C  * SYSTEM RELIABILITY.                                         *
C  * THE ABOVE PROCESS IS REPEATED FOR NOLMC MONTE CARLO RUNS, WITH*
C  * COUNTERS FOR EACH SYSTEM TO TRACK THE NUMBER OF TIMES THAT THE*
C  * CONFIDENCE INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.    *
C  *                                                             *
C  ***************************************************************
C
      DIMENSION CC(550),THETA(550),EK(550)
      DIMENSION  ARCHAT(2000,5), ATRS(4), BI(12,15), C(4,4), DEV
     1  (1000), PARAM(3,5), R(500,5), RC(1000,5), RCHAT
     2   (5), RLBS(12),RS(1000), SIG(12,15), SIGSQ
     3  (12,15), SMSZ(15), TEMP(200), TRC(5), TRS(1),VRH(10,11),
     4  WK(200), WKAREA(10), X(2)
      DOUBLE       PRECISIONDSEED
      REAL         KHAT
      DATA     RLBS / .5, .55, .6, .65, .7, .75, .8, .85, .9, .925, .95,
     1  .98 /
C
      DATA     SMSZ / 8., 9., 10., 11., 12., 13., 14., 15., 20., 25.,
     1  30., 40., 50., 75., 100. /
C
C
C  INITIALIZE VARIABLES
      PI         =4. * ATAN(1.)
      READ*,DSEED,T
```

```
      CALL RANSET (T)
      TIME      =100.
      NRLBS     =12
      NSMSZ     =15
      NSMSZ1    =NSMSZ - 1
      NRLBS1    =NRLBS - 1
      ISD       =12
      NTRUNC    =0
      LTRUNC=0
      NS1C99    =0
      NS1C95    =0
      NS1C90    =0
      NS1C80    =0
      NS1C70    =0
      NS1C60    =0
      NS1C50    =0
      NS2C99    =0
      NS2C95    =0
      NS2C90    =0
      NS2C80    =0
      NS2C70    =0
      NS2C60    =0
      NS2C50    =0
      NS3C99    =0
      NS3C95    =0
      NS3C90    =0
      NS3C80    =0
      NS3C70    =0
      NS3C60    =0
      NS3C50    =0
      NS4C99=0
      NS4C95    =0
      NS4C90    =0
      NS4C80    =0
      NS4C70    =0
      NS4C60    =0
      NS4C50    =0
C
      PRINT  409
C  READ THE SAMPLE SIZE AND NUMBER OF MONTE CARLO RUNS
      READ*,NMCT,IDIMT,NPTS
      READ*,M1,M2,M3,M4,M5,M6,M7
      READ *, NSAM, NOLMC
      PRINT  419, NSAM, NOLMC
      RNSAM     =NSAM
      M=NSAM
      MR=0
C  READ THE TRUE COMPONENT PARAMETERS
      READ *, ((PARAM(I, J), I=1, 3), J=1, 5)
      PRINT  429,  ((J, (PARAM(I, J), I=1, 3), CMPREL(TIME,PARAM(1, J),
     1  PARAM(2, J), PARAM(3, J))), J=1, 5)
      READ*,((SIG(I,J),J=1,15), I=1,12)
      READ*,((BI(I,J),J=1,15),I=1,12)
      PRINT  439, ((SIG(I, J), J=1, 15), I=1,12)
      PRINT  439,  ((BI(I, J), J=1, 15), I=1,12)
C
C   ***************************************************************************
```

```
C    ***********************************************************************
C   THIS OUTSIDE LOOP FROM HERE TO STATEMENT 300 COMPLETES
C   NOLMC MONTE CARLO RUNS OF THE SIMULATION.
C
          DO 300 NCOUNT = 1, NOLMC
C
C    ***********************************************************************
C
C   FOR EACH OF 5 COMPONENTS, THIS LOOP GENERATES THE MLE OF RCHAT
C   FROM THE MLES OF K AND THETA.  IT DETERMINES THE VARIANCE OF
C  RCHAT AND THEN SAMPLES FROM THE NORMAL DISTRIBUTION FOR NDEV
C   SAMPLE RELIABILITIES OF EACH COMPONENT.
C
          DO 290 J =1, 5
          TK        = PARAM(1, J)
          TTHETA    = PARAM(2, J)
          TC        = PARAM(3, J)
C   PASS INFORMATION ON COMPONENT NUMBER AND
C   SAMPLE SIZE THRU ARRAY R
          R(205, 1) = J
          R(204, J) = NSAM
C   DETERMINE THE TRUE COMPONENT RELIABILITY
          TRC(J)    =CMPREL(TIME, TK, TTHETA, TC)
C
C   GATHER NSAM FAILURE TIMES FROM THE
C   WEIBULL DISTRIBUTION WITH TRUE PARAMETERS
C   TK,TTHETA, AND TC
C
          CALL GGWIB (DSEED, TK, NSAM, TEMP)
            DO 20 I   =1, NSAM
            R(I, J)   =TTHETA * TEMP(I) + TC
    20      CONTINUE
C   DETERMINE THE LIKELIHOOD OF DRAWING THIS SAMPLE
C   USING THE TRUE K, THETA, AND C
C
C   DETERMINE THE MLE OF THETA AND K
C   BY HARTER & MOORES ITERATIVE SCHEME
C
       CC(1)=TC
       THETA(1)=TTHETA
       EK(1)=TK
       CALL PARES(NSAM,M,CC,THETA,EK,MR,THAT,KHAT,R(1,J))
C
C   DETERMINE RCHAT, THE MLE OF THE COMPONENT RELIABILITY, USING
C   THE TRUE C AND THE MLES OF K AND THETA
          RCHAT(J)  = CMPREL(TIME, KHAT, THAT, TC)
C
C   GIVEN THE MLE OF THE COMPONENT RELIABILITY AND THE SAMPLE SIZE,
C   ENTER THE 2-DIMENSIONAL ARRAY SI AND FIND THE BIAS OF THE
C   ESTIMATOR
          DO 50 I   =1, NSMSZ1
          LSMSZ     =I
          IF (RNSAM .LE. SMSZ(I+1)) GO TO 60
    50    CONTINUE
          GO TO 90
    60    IF (RCHAT(J) .LT. .5) GO TO 100
```

```
            DO 70 I    =1, NRLBS1
            LRLBS      =I
            IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 80
   70       CONTINUE
            GO TO 90
   80       CALL IBCICU (BI, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ,
     1        C, WK, IER)
            CALL IBCEVU (RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ, C,
     1        RCHAT(J), RNSAM, BIAS, IER)
            IF (LRLBS .LT. NRLBS) GO TO 110
   90       BIAS     =0.
            GO TO 110
  100       IF (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ = LSMSZ + 1
            BIAS     = BI(1, LSMSZ)
  110       RCHAT(J)  =RCHAT(J) - BIAS
            ARCHAT(NCOUNT, J) = RCHAT(J)
C
C  GIVEN THE UNBIASED ESTIMATOR OF THE COMPONENT RELIBILITY,
C  ENTER THE 2-DIMENSIONAL ARRAY SIG AND FIND THE STANDARD
C  DEVIATION OF THE UNBIASED ESTIMATOR
            Z          =RCHAT(J)
            RCRLB      =SQRT(Z**2*(ALOG(Z))**2*(1.109-.514*
     1        ALOG(-ALOG(Z)) + .608*(ALOG(-ALOG(Z)))**2)/RNSAM)
            DO 120 I   =1, NSMSZ1
            LSMSZ      =I
            IF (RNSAM .LE. SMSZ(I+1)) GO TO 130
  120       CONTINUE
            GO TO 160
  130       IF (RCHAT(J) .LT. .5) GO TO 170
              DO 140 I  =1, NRLBS1
              LRLBS     =I
              IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 150
  140         CONTINUE
            GO TO 160
  150       CALL IBCICU (SIG, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS,
     1        LSMSZ, C, WK, IER)
            CALL IBCEVU (RLBS, NRLBS, SMSZ,NSMSZ, LRLBS, LSMSZ, C,
     1        RCHAT(J), RNSAM, SIGMA, IER)
            IF (SIGMA .GE. RCRLB) GO TO 180
  160       SIGMA      =RCRLB
            IER        =0
            GO TO 180
  170       IF (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ=LSMSZ + 1
            SIGMA      =SIG(1, LSMSZ)
            IF (SIGMA .GE. RCRLB) GO TO 180
            SIGMA      =RCRLB
  180       CONTINUE
C
C    FORM A VECTOR OF NDEV SAMPLE RELIABILITIES WITH MEAN RCHAT
C    AND A BETA(ALPHA,BETA) DISTRIBUTION
        NDEV=NMCT
             Q     = RCHAT(J)/(1.-RCHAT(J))
             BETA= Q/(((1.+Q)**3)*SIGMA**2) -1./(1.+Q)
             ALPHA= Q*BETA
             CALL GGBTR(DSEED,ALPHA,BETA,NDEV,DEV)
        DO 190 I=1,NDEV
               RC(I,J)=DEV(I)
```

```
      IF(RC(I,J).LT.0.0) LTRUNC=LTRUNC+1
      IF(RC(I,J).LT.0.0) RC(I,J)=0.0
C  TRUNCATE THE  BETA  DISTRIBUTION IF COMPONENT RELIABILITY
C  IS GREATER THAN 1
            IF (RC(I,J) .LE. 1.) GO TO 190
            RC(I,J)  =1.
            NTRUNC   =NTRUNC + 1
  190 CONTINUE
  290     CONTINUE
C  ***************************************************************
C
C  FOR EACH OF THE 4 SYSTEMS, THE DIFFERENT COMPONENTS ARE COMBINED
C  TO YIELD NDEV SAMPLES OF THE SYSTEM RELIABILITY.  THESE SAMPLES
C  ARE SEQUENCED IN ASCENDING ORDER AND THEN COUNTERS KEEP TRACK
C  OF WHEN THE 99, 95, 90, 80, 70, 60, AND 50 PERCENT CONFIDENCE
C  INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.
C
C  SYSTEM 1
            NMC      =1
            IDIM     =1
            CALL REL1 (NMC, TRC, TRS, IDIM)
            ATRS(1)  =TRS(1)
         NMC=NMCT
         IDIM=IDIMT
            CALL REL1 (NMC, RC, RS, IDIM)
            CALL VSRTA (RS, IDIM)
         IF(RS(M1).LE.TRS(1)) NS1C99=NS1C99+1
         IF(RS(M2).LE.TRS(1)) NS1C95=NS1C95+1
         IF(RS(M3).LE.TRS(1)) NS1C90=NS1C90+1
         IF(RS(M4).LE.TRS(1)) NS1C80=NS1C80+1
         IF(RS(M5).LE.TRS(1)) NS1C70=NS1C70+1
         IF(RS(M6).LE.TRS(1)) NS1C60=NS1C60+1
         IF(RS( M7).LE.TRS(1)) NS1C50=NS1C50+1
C
C  SYSTEM 2
            NMC      =1
            IDIM     =1
            CALL REL2 (NMC, TRC, TRS, IDIM)
            ATRS(2)  =TRS(1)
         NMC=NMCT
         IDIM=IDIMT
            CALL REL2 (NMC, RC, RS, IDIM)
            CALL VSRTA (RS, IDIM)
         IF(RS(M1).LE.TRS(1)) NS2C99=NS2C99+1
         IF(RS(M2).LE.TRS(1)) NS2C95=NS2C95+1
         IF(RS(M3).LE.TRS(1)) NS2C90=NS2C90+1
         IF(RS(M4).LE.TRS(1)) NS2C80=NS2C80+1
         IF(RS(M5).LE.TRS(1)) NS2C70=NS2C70+1
         IF(RS(M6).LE.TRS(1)) NS2C60=NS2C60+1
         IF(RS( M7).LE.TRS(1)) NS2C50=NS2C50+1
C
C  SYSTEM 3
           NMC      = 1
           IDIM     = 1
           CALL REL3 (NMC, TRC, TRS, IDIM)
           ATRS(3)  = TRS(1)
         NMC=NMCT
```

```
      IDIM=IDIMT
         CALL REL3 (NMC, RC, RS, IDIM)
         CALL VSRTA (RS, IDIM)
      IF(RS(M1).LE.TRS(1)) NS3C99=NS3C99+1
      IF(RS(M2).LE.TRS(1)) NS3C95=NS3C95+1
      IF(RS(M3).LE.TRS(1)) NS3C90=NS3C90+1
      IF(RS(M4).LE.TRS(1)) NS3C80=NS3C80+1
      IF(RS(M5).LE.TRS(1)) NS3C70=NS3C70+1
      IF(RS(M6).LE.TRS(1)) NS3C60=NS3C60+1
      IF(RS( M7).LE.TRS(1)) NS3C50=NS3C50+1
C
C  SYSTEM 4
         NMC        = 1
         IDIM       = 1
         CALL REL4 (NMC, TRC, TRS, IDIM)
         ATRS(4)    = TRS(1)
      NMC=NMCT
      IDIM=IDIMT
         CALL REL4 (NMC, RC, RS, IDIM)
         CALL VSRTA (RS, IDIM)
      IF(RS(M1).LE.TRS(1)) NS4C99=NS4C99+1
      IF(RS(M2).LE.TRS(1)) NS4C95=NS4C95+1
      IF(RS(M3).LE.TRS(1)) NS4C90=NS4C90+1
      IF(RS(M4).LE.TRS(1)) NS4C80=NS4C80+1
      IF(RS(M5).LE.TRS(1)) NS4C70=NS4C70+1
      IF(RS(M6).LE.TRS(1)) NS4C60=NS4C60+1
      IF(RS( M7).LE.TRS(1)) NS4C50=NS4C50+1
  300    CONTINUE
C
C    ***************************************************************************
C    ***************************************************************************
C
      NPT=5*NPTS*NOLMC
      PRINT 350, LTRUNC
  350  FORMAT(1H ,11HTHERE WERE ,I7,18H LOWER TRUNCATIONS)
       PRINT 469,NTRUNC,NPT
       RNOLMC     = NOLMC
C  FOR SYSTEM 1, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
       PS1C99     = NS1C99 / RNOLMC
       PS1C95     = NS1C95 / RNOLMC
       PS1C90     = NS1C90 / RNOLMC
       PS1C80     = NS1C80 / RNOLMC
       PS1C70     = NS1C70 / RNOLMC
       PS1C60     = NS1C60 / RNOLMC
       PS1C50     = NS1C50 / RNOLMC
C  FOR SYSTEM 2, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
       PS2C99     = NS2C99 / RNOLMC
       PS2C95     = NS2C95 / RNOLMC
       PS2C90     = NS2C90 / RNOLMC
       PS2C80     = NS2C80 / RNOLMC
       PS2C70     = NS2C70 / RNOLMC
       PS2C60     = NS2C60 / RNOLMC
       PS2C50     = NS2C50 / RNOLMC
C  FOR SYSTEM 3, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
```

```
      PS3C99    = NS3C99 / RNOLMC
      PS3C95    = NS3C95 / RNOLMC
      PS3C90    = NS3C90 / RNOLMC
      PS3C80    = NS3C80 / RNOLMC
      PS3C70    = NS3C70 / RNOLMC
      PS3C60    = NS3C60 / RNOLMC
      PS3C50    = NS3C50 / RNOLMC
C  FOR SYSTEM 4, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS4C99    = NS4C99 / RNOLMC
      PS4C95    = NS4C95 / RNOLMC
      PS4C90    = NS4C90 / RNOLMC
      PS4C80    = NS4C80 / RNOLMC
      PS4C70    = NS4C70 / RNOLMC
      PS4C60    = NS4C60 / RNOLMC
      PS4C50    = NS4C50 / RNOLMC
      PRINT  479
      PRINT 489,ATRS(1),PS1C99,PS1C95,PS1C90,PS1C80,PS1C70,PS1C60,PS1C50
       PRINT 499
      PRINT 489,ATRS(2),PS2C99,PS2C95,PS2C90,PS2C80,PS2C70,PS2C60,PS2C50
       PRINT  509
      PRINT 489,ATRS(3),PS3C99,PS3C95,PS3C90,PS3C80,PS3C70,PS3C60,PS3C50
       PRINT  519
      PRINT 489,ATRS(4),PS4C99,PS4C95,PS4C90,PS4C80,PS4C70,PS4C60,PS4C50
C
C   ***************************************************************
C   ***************************************************************
C
      STOP "FCRAWHILE"
C
  409  FORMAT  ( 1H1  )
  419  FORMAT  (  " ************************************************************"004
     1   "**********",  /  ,  " *", T62,  "*",  /  ,  " *", T62,  "*",  / 0
     2   ,  " *", T25,  "SAMPLE "  "SIZE = ", I3, T62,  "*",  /  ,  " *",004
     3   T62,  "*",  /  ,  " *", T22,  "MONTE CARLO "  "SIZE = ", I3,     004
     4   T62,  "*",  /  ,  " *", T62,  "*",  /  ,  " *", T62,  "*",  /  ,  0046
     5   " ************************************************************"
     6   "*",  /  /  /  /  /  /  /  )
  429  FORMAT  ( 5(1X,  "COMPONENT ", I1,  /  , 6X,  "K = ", F4.2,  /  ,  004C
     1   6X,  "THETA = ", F5.0,  /  , 6X,  "C = ", F2.0,  /  , 6X,
     2   "RELIABILITY =", F7.5,  /  / ) )
  439  FORMAT  ( 15(1X, F7.5) )
  449  FORMAT  (/," **********",/," MC=",I4,4X,"J =",I2,4X,"NSTART =",I2
     1                          ,/,  " KHAT=" E13.6, 5X "THAT=" E13.6 /004
     2   " ITERATIONS=" I3, 5X "IER=" I3 /  " **********"  )              0041
  459  FORMAT  (  /  " ********************" /  /  " DID NOT CONVERGE IN"0
     1   " 8 ATTEMPTS  WITH DIFFERENT STARTING KHATS" /  " THEREFORE GO"004
     2   "ING ON TO THE NEXT   COMPONENT" /  /  " ********************" 0
     3   )
  469 FORMAT ( / / , " THERE WERE", I7, " HIGH TRUNCATIONS OUT OF"
     1  , I8, " RELIABILITY DEVIATES", / / )
  479  FORMAT  (  /  " ***** SYSTEM 1 *****" /  " (3 COMPONENTS IN SERI"00
     1   "ES)"  )
  489  FORMAT  (  / ,  " TRUE SYSTEM RELIABILITY =", F7.5,  /
     1   " THE 99 PERCENT CONFIDENCE INTERVAL COVERED ",F6.4,
     2   " OF THE RUNS",/,
     3   " THE 95 PERCENT CONFIDENCE INTERVAL COVERED ", F6.4,
```

```
     4   " OF THE RUNS",/," THE 90 PERCENT CONFIDENCE INTERVAL COVERED "
     5 , F6.4," OF THE RUNS",/," THE 80 PERCENT CONFIDENCE INTERVAL"
     6   " COVERED ",F6.4," OF THE RUNS",/," THE 70 PERCENT CONFIDENCE "
     7   "INTERVAL COVERED ",F6.4," OF THE RUNS",/," THE 60 PERCENT "
     8   "CONFIDENCE INTERVAL COVERED ",F6.4," OF THE RUNS",/," THE",
     9   " 50 PERCENT CONFIDENCE INTERVAL COVERED ",F6.4," OF THE RUNS",
     1   ///)
   499  FORMAT  (  /  " ***** SYSTEM 2 *****" /  " (1 COMPONENT IN SERIE"0
     1   "S WITH 2 "  "IN PARALLEL)"  )
   509  FORMAT  (  /  " ***** SYSTEM 3 *****" /  " (3 COMPONENTS IN PARA"00434C
     1   "LLEL)"  )
   519  FORMAT  (  /  " ***** SYSTEM 4 *****" /  " (A 5-COMPONENT COMPLE"00
     1   "X NETWORK)"  )
   529  FORMAT  ( 5(1X, F10.8, 5X) )                                    0043
C
       END
```

D. Adjusted Univariate-Normal Technique: Notes and Computer Program Listing

This appendix contains the computer program listing for the adjusted univariate-normal technique, i.e., the univariate-normal technique with Gatliffe's method of adjusting for perfect system reliability estimates incorporated.

It should be noted that due to large execution times for this program, 100 Monte Carlo simulations were performed in each of six computer runs for each component sample size. The results were then totaled and averaged over the six runs for each component sample size. The execution times for 100 Monte Carlo simulations are:

Component Sample Size	Execution Time (Seconds)
10	247
15	259
20	270
50	332
100	433

The adjusted univariate-normal technique fits inside 140,000 words of core on the CDC 6600 machine as does the other univariate-normal technique.

```
      PROGRAM WEIB(INPUT,OUTPUT)
C
C  ****************************************************************
C  *                                                              *
C  * FOR EACH COMPONENT OF A COMPLEX SYSTEM, THIS PROGRAM         *
C  * GENERATES A SAMPLE OF SIZE NSAM FROM THE WEIBULL DISTRIBUTION *
C  * USING THE TRUE PARAMETERS TK (SHAPE), TTHETA (SCALE), AND TC *
C  * (LOCATION).  FROM THIS SAMPLE, THE MAXIMUM LIKELIHOOD        *
C  * ESTIMATORS (MLES) FOR K AND THETA ARE DERIVED USING HARTER & *
C  * MOORES ITERATIVE SCHEME AND THE SIMULTANEOUS EQUATION SOLVER *
C  * ZSYSTM.  THE MLES ARE COMBINED TO YIELD RCHAT, THE MLE FOR THE*
C  * COMPONENT RELIABILITY.  GIVEN THE RELIABILITY AND THE SAMPLE *
C  * SIZE NSAM, RCHAT IS ASYMPTOTICALLY NORMALLY DISTRIBUTED WITH A*
C  * SPECIFIED VARIANCE.  THEREFORE WE CAN SAMPLE FROM THE NORMAL *
C  * DISTRIBUTION TO OBTAIN A VECTOR OF SAMPLE COMPONENT          *
C  *RELIABILITIES.  THIS PROCESS IS REPEATED FOR EACH COMPONENT   *
C  * AND THEN THE RELIABILITIES ARE COMBINED FOR 4 DIFFERENT TYPES*
C  * OF SYSTEMS TO YIELD 4 VECTORS OF SAMPLE SYSTEM RELIABILITIES. *
C  * THESE VECTORS ARE ORDERED AND THEN THE 95, 90, AND 80 PERCENT *
C  * LOWER CONFIDENCE LIMITS ARE PICKED.  THIS ESTABLISHES THE 95, *
C  * 90, AND 80 PERCENT CONFIDENCE INTERVALS FOR EACH SYSTEM AND IT*
C  * IS NOTED WHETHER EACH OF THESE INTERVALS CONTAINS THE TRUE    *
C  * SYSTEM RELIABILITY.                                          *
C  * THE ABOVE PROCESS IS REPEATED FOR NOLMC MONTE CARLO RUNS, WITH*
C  * COUNTERS FOR EACH SYSTEM TO TRACK THE NUMBER OF TIMES THAT THE*
C  * CONFIDENCE INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.     *
C  *                                                              *
C  ****************************************************************
C
      DIMENSION CC(550),THETA(550),EK(550)
      DIMENSION  ARCHAT(2000,5), ATRS(4), BI(12,15), C(4,4), DEV
     1  (600), PARAM(3,5), R(500,5),RC(600,5),RCHAT
     2  (5), RLBS(12), RS(600), SIG(12,15), SIGSQ
     3  (12,15), SMSZ(15), TEMP(200), TRC(5), TRS(1),
     4  WK(200)
      DOUBLE      PRECISIONDSEED
      REAL        KHAT
      DATA    RLBS / .5, .55, .6, .65, .7, .75, .8, .85, .9, .925, .95,
     1  .98 /
C
      DATA    SMSZ / 8., 9., 10., 11., 12., 13., 14., 15., 20., 25.,
     1  30., 40., 50., 75., 100. /
C
C  INITIALIZE VARIABLES
      PI        =4. * ATAN(1.)
      READ*,DSEED,T
      CALL RANSET (T)
      TIME      =100.
      NRLBS     =12
      NSMSZ     =15
      NSMSZ1    =NSMSZ - 1
      NRLBS1    =NRLBS - 1
      ISD       =12
      NTRUNC    =0
      LTRUNC=0
      NS1C99    =0
      NS1C95    =0
```

```
      NS1C90     =0
      NS1C80     =0
      NS1C70     =0
      NS1C60     =0
      NS1C50     =0
      NS2C99     =0
      NS2C95     =0
      NS2C90     =0
      NS2C80     =0
      NS2C70     =0
      NS2C60     =0
      NS2C50     =0
      NS3C99     =0
      NS3C95     =0
      NS3C90     =0
      NS3C80     =0
      NS3C70     =0
      NS3C60     =0
      NS3C50     =0
      NS4C99=0
      NS4C95     =0
      NS4C90     =0
      NS4C80     =0
      NS4C70     =0
      NS4C60     =0
      NS4C50     =0
C
      PRINT  409
C  READ THE SAMPLE SIZE AND NUMBER OF MONTE CARLO RUNS
      READ*,NUM
      READ*,NMCT,IDIMT,NPTS
      READ*,M1,M2,M3,M4,M5,M6,M7
      READ *, NSAM, NOLMC
      PRINT  419, NSAM, NOLMC
      RNSAM      =NSAM
      M=NSAM
      MR=0
C  READ THE TRUE COMPONENT PARAMETERS
      READ *, ((PARAM(I, J), I=1, 3), J=1, 5)
      PRINT  429,  ((J, (PARAM(I, J), I=1, 3), CMPREL(TIME,PARAM(1, J)
     1  PARAM(2, J), PARAM(3, J))), J=1, 5)
      READ*,((SIG(I,J),J=1,15), I=1,12)
      READ*,((EI(I,J),J=1,15),I=1,12)
      PRINT  439, ((SIG(I, J), J=1, 15), I=1,12)
      PRINT  439,  ((BI(I, J), J=1, 15), I=1,12)
C
C   ***************************************************************
C   ***************************************************************
C  THIS OUTSIDE LOOP FROM HERE TO STATEMENT 300 COMPLETES
C  NOLMC MONTE CARLO RUNS OF THE SIMULATION.
C
      DO 300 NCOUNT = 1, NOLMC
C
C   ***************************************************************
C
C  FOR EACH OF 5 COMPONENTS, THIS LOOP GENERATES THE MLE OF RCHAT
C  FROM THE MLES OF K AND THETA.  IT DETERMINES THE VARIANCE OF
```

```
C RCHAT AND THEN SAMPLES FROM THE NORMAL DISTRIBUTION FOR NDEV
C  SAMPLE RELIABILITIES OF EACH COMPONENT.
C
      DO 290 J =1, 5
      TK       = PARAM(1, J)
      TTHETA   = PARAM(2, J)
      TC       = PARAM(3, J)
C  PASS INFORMATION ON COMPONENT NUMBER AND
C  SAMPLE SIZE THRU ARRAY R
      R(205, 1) = J
      R(204, J) = NSAM
C  DETERMINE THE TRUE COMPONENT RELIABILITY
      TRC(J)    =CMPREL(TIME, TK, TTHETA, TC)
C
C  GATHER NSAM FAILURE TIMES FROM THE
C  WEIBULL DISTRIBUTION WITH TRUE PARAMETERS
C  TK,TTHETA, AND TC
C
      CALL GGWIB (DSEED, TK, NSAM, TEMP)
        DO 20 I   =1, NSAM
        R(I, J)   =TTHETA * TEMP(I) + TC
   20   CONTINUE
C  DETERMINE THE LIKELIHOOD OF DRAWING THIS SAMPLE
C  USING THE TRUE K, THETA, AND C
C
C  DETERMINE THE MLE OF THETA AND K
C  BY HARTER & MOORES ITERATIVE SCHEME
C
      CC(1)=TC
      THETA(1)=TTHETA
      EK(1)=TK
      CALL PARES(NSAM,M,CC,THETA,EK,MR,THAT,KHAT,R(1,J))
C
C  DETERMINE RCHAT, THE MLE OF THE COMPONENT RELIABILITY, USING
C  THE TRUE C AND THE MLES OF K AND THETA
        RCHAT(J)  = CMPREL(TIME, KHAT, THAT, TC)
C
C  GIVEN THE MLE OF THE COMPONENT RELIABILITY AND THE SAMPLE SIZE,
C  ENTER THE 2-DIMENSIONAL ARRAY BI AND FIND THE BIAS OF THE
C  ESTIMATOR
        DO 50 I   =1, NSMSZ1
        LSMSZ     =I
        IF (RNSAM .LE. SMSZ(I+1)) GO TO 60
   50   CONTINUE
        GO TO 90
   60   IF (RCHAT(J) .LT. .5) GO TO 100
        DO 70 I   =1, NRLBS1
        LRLBS     =I
        IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 80
   70   CONTINUE
        GO TO 90
   80   CALL IBCICU (BI, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ,
     1    C, HK, IER)
        CALL IBCEVU (RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ, C,
     1    RCHAT(J), RNSAM, BIAS, IER)
        IF (LRLBS .LT. NRLBS) GO TO 110
```

```
   90    BIAS    =0.
           GO TO 110
  100    IF (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ = LSMSZ + 1
         BIAS    = BI(1, LSMSZ)
  110      RCHAT(J)  =RCHAT(J) - BIAS
           ARCHAT(NCOUNT, J) = RCHAT(J)
C
C  GIVEN THE UNBIASED ESTIMATOR OF THE COMPONENT RELIBILITY,
C  ENTER THE 2-DIMENSIONAL ARRAY SIG AND FIND THE STANDARD
C  DEVIATION OF THE UNBIASED ESTIMATOR
           Z          =RCHAT(J)
           RCRLB      =SQRT(Z**2*(ALOG(Z))**2*(1.109-.514*
     1       ALOG(-ALOG(Z)) + .608*(ALOG(-ALOG(Z)))**2)/RNSAM)
             DO 120 I  =1, NSMSZ1
             LSMSZ     =I
             IF (RNSAM .LE. SMSZ(I+1)) GO TO 130
  120        CONTINUE
           GO TO 160
  130      IF (RCHAT(J) .LT. .5) GO TO 170
             DO 140 I  =1, NRLBS1
             LRLBS     =I
             IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 150
  140        CONTINUE
           GO TO 160
  150      CALL IBCICU (SIG, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS,
     1       LSMSZ, C, WK, IER)
           CALL IBCEVU (RLBS, NRLBS, SMSZ,NSMSZ, LRLBS, LSMSZ, C,
     1       RCHAT(J), RNSAM, SIGMA, IER)
           IF (SIGMA .GE. RCRLB) GO TO 180
  160      SIGMA     =RCRLB
           IER       =0
           GO TO 180
  170      IF (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ=LSMSZ + 1
           SIGMA     =SIG(1, LSMSZ)
           IF (SIGMA .GE. RCRLB) GO TO 180
           SIGMA     =RCRLB
  180      CONTINUE
C
C  FORM A VECTOR OF NDEV SAMPLE RELIABILITIES WITH MEAN RCHAT
C  AND A STANDARD DEVIATION OF SIGMA
         NDEV=NMCT
           CALL GGNML (DSEED, NDEV, DEV)
         DO 190 I=1,NDEV
             RC(I, J)  =RCHAT(J) + DEV(I) * SIGMA
         IF(RC(I,J).LT.0.0) LTRUNC=LTRUNC+1
         IF(RC(I,J).LT.0.0) RC(I,J)=0.0
C  TRUNCATE THE NORMAL DISTRIBUTION IF COMPONENT RELIABILITY
C  IS GREATER THAN 1
             IF (RC(I,J) .LE. 1.) GO TO 190
             RC(I,J)  =1.
             NTRUNC   =NTRUNC + 1
  190 CONTINUE
  290    CONTINUE
C  ********************************************************************
C
C  FOR EACH OF THE 4 SYSTEMS, THE DIFFERENT COMPONENTS ARE COMBINED
C  TO YIELD NDEV SAMPLES OF THE SYSTEM RELIABILITY.  THESE SAMPLES
```

```
C  ARE SEQUENCED IN ASCENDING ORDER AND THEN COUNTERS KEEP TRACK
C  OF WHEN THE 99, 95, 90, 80, 70, 60, AND 50 PERCENT CONFIDENCE
C  INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.
C
C  SYSTEM 1
        NMC      =1
        IDIM     =1
        CALL REL1 (NMC, TRC, TRS, IDIM)
        ATRS(1)  =TRS(1)
      NMC=NMCT
      IDIM=IDIMT
      NALPHA=1
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS(M1).LE.TRS(1)) NS1C99=NS1C99+1
      NALPHA=2
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS(M2).LE.TRS(1)) NS1C95=NS1C95+1
      NALPHA=3
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS(M3).LE.TRS(1)) NS1C90=NS1C90+1
      NALPHA=4
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS(M4).LE.TRS(1)) NS1C80=NS1C80+1
      NALPHA=5
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS(M5).LE.TRS(1)) NS1C70=NS1C70+1
      NALPHA=6
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS(M6).LE.TRS(1)) NS1C60=NS1C60+1
      NALPHA=7
      CALL RELG1(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, IDIM)
      IF(RS( M7).LE.TRS(1)) NS1C50=NS1C50+1
C
C  SYSTEM 2
        NMC      =1
        IDIM     =1
        CALL REL2 (NMC, TRC, TRS, IDIM)
        ATRS(2)  =TRS(1)
      NMC=NMCT
      IDIM=IDIMT
      NALPHA=1
      CALL RELG2(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, ICIM)
      IF(RS(M1).LE.TRS(1)) NS2C99=NS2C99+1
      NALPHA=2
      CALL RELG2(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
        CALL VSRTA (RS, ICIM)
      IF(RS(M2).LE.TRS(1)) NS2C95=NS2C95+1
      NALPHA=3
      CALL RELG2(NMC,RC,RS,ICIM,NUM,NALPHA,NSAM)
```

AD-A094 810 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 12/1
A COMPARISON OF THE ACCURACY OF UNIVARIATE AND BIVARIATE TECHNI--ETC(U)
DEC 80 K M DEPUY
UNCLASSIFIED AFIT/GOR/OS/80D-2 NL

3 OF 3
AD A094810

```
          CALL VSRTA (RS, IDIM)
        IF(RS(M3).LE.TRS(1)) NS2C90=NS2C90+1
        NALPHA=4
        CALL RELG2(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M4).LE.TRS(1)) NS2C80=NS2C80+1
        NALPHA=5
        CALL RELG2(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M5).LE.TRS(1)) NS2C70=NS2C70+1
        NALPHA=6
        CALL RELG2(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M6).LE.TRS(1)) NS2C60=NS2C60+1
        NALPHA=7
        CALL RELG2(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS( M7).LE.TRS(1)) NS2C50=NS2C50+1
C
C  SYSTEM 3
          NMC       = 1
          IDIM      = 1
          CALL REL3 (NMC, TRC, TRS, IDIM)
          ATRS(3)   = TRS(1)
        NMC=NMCT
        IDIM=IDIMT
        NALPHA=1
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M1).LE.TRS(1)) NS3C99=NS3C99+1
        NALPHA=2
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M2).LE.TRS(1)) NS3C95=NS3C95+1
        NALPHA=3
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M3).LE.TRS(1)) NS3C90=NS3C90+1
        NALPHA=4
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M4).LE.TRS(1)) NS3C80=NS3C80+1
        NALPHA=5
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M5).LE.TRS(1)) NS3C70=NS3C70+1
        NALPHA=6
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS(M6).LE.TRS(1)) NS3C60=NS3C60+1
        NALPHA=7
        CALL RELG3(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
          CALL VSRTA (RS, IDIM)
        IF(RS( M7).LE.TRS(1)) NS3C50=NS3C50+1
C
C  SYSTEM 4
          NMC       = 1
```

```
            IDIM      = 1
            CALL REL4 (NMC, TRC, TRS, IDIM)
            ATRS(4)   = TRS(1)
          NMC=NMCT
          IDIM=IDIMT
          NALPHA=1
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
             CALL VSRTA (RS, IDIM)
          IF(RS(M1).LE.TRS(1)) NS4C99=NS4C99+1
          NALPHA=2
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
             CALL VSRTA (RS, IDIM)
          IF(RS(M2).LE.TRS(1)) NS4C95=NS4C95+1
          NALPHA=3
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
             CALL VSRTA (RS, IDIM)
          IF(RS(M3).LE.TRS(1)) NS4C90=NS4C90+1
          NALPHA=4
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
             CALL VSRTA (RS, IDIM)
          IF(RS(M4).LE.TRS(1)) NS4C80=NS4C80+1
          NALPHA=5
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
             CALL VSRTA (RS, IDIM)
          IF(RS(M5).LE.TRS(1)) NS4C70=NS4C70+1
          NALPHA=6
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
              CALL VSRTA (RS, IDIM)
          IF(RS(M6).LE.TRS(1)) NS4C60=NS4C60+1
          NALPHA=7
          CALL RELG4(NMC,RC,RS,IDIM,NUM,NALPHA,NSAM)
              CALL VSRTA (RS, IDIM)
          IF(RS( M7).LE.TRS(1)) NS4C50=NS4C50+1
      300 CONTINUE
C
C   ***************************************************************0
C   ***************************************************************
C
          NPT=5*NPTS*NOLMC
          PRINT 350, LTRUNC
      350 FORMAT(1H ,11HTHERE WERE ,I7,18H LOWER TRUNCATIONS)
           PRINT 469,NTRUNC,NPT
           RNOLMC    = NOLMC
C  FOR SYSTEM 1, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
           PS1C99    = NS1C99 / RNOLMC
           PS1C95    = NS1C95 / RNOLMC
           PS1C90    = NS1C90 / RNOLMC
           PS1C80    = NS1C80 / RNOLMC
           PS1C70    = NS1C70 / RNOLMC
           PS1C60    = NS1C60 / RNOLMC
           PS1C50    = NS1C50 / RNOLMC
C  FOR SYSTEM 2, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
           PS2C99    = NS2C99 / RNOLMC
           PS2C95    = NS2C95 / RNOLMC
           PS2C90    = NS2C90 / RNOLMC
```

```
      PS2C80    = NS2C80 / RNOLMC
      PS2C70    = NS2C70 / RNOLMC
      PS2C60    = NS2C60 / RNOLMC
      PS2C50    = NS2C50 / RNOLMC
C  FOR SYSTEM 3, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS3C99    = NS3C99 / RNOLMC
      PS3C95    = NS3C95 / RNOLMC
      PS3C90    = NS3C90 / RNOLMC
      PS3C80    = NS3C80 / RNOLMC
      PS3C70    = NS3C70 / RNOLMC
      PS3C60    = NS3C60 / RNOLMC
      PS3C50    = NS3C50 / RNOLMC
C  FOR SYSTEM 4, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS4C99    = NS4C99 / RNOLMC
      PS4C95    = NS4C95 / RNOLMC
      PS4C90    = NS4C90 / RNOLMC
      PS4C80    = NS4C80 / RNOLMC
      PS4C70    = NS4C70 / RNOLMC
      PS4C60    = NS4C60 / RNOLMC
      PS4C50    = NS4C50 / RNOLMC
      PRINT  479
      PRINT 489,ATRS(1),PS1C99,PS1C95,PS1C90,PS1C80,PS1C70,PS1C60,PS1C50
       PRINT 499
      PRINT 489,ATRS(2),PS2C99,PS2C95,PS2C90,PS2C80,PS2C70,PS2C60,PS2C50
       PRINT  509
      PRINT 489,ATRS(3),PS3C99,PS3C95,PS3C90,PS3C80,PS3C70,PS3C60,PS3C50
       PRINT  519
      PRINT 489,ATRS(4),PS4C99,PS4C95,PS4C90,PS4C80,PS4C70,PS4C60,PS4C50
C
C   ***************************************************************
C   ***************************************************************
C
      STOP "FORAWHILE"
C
  409 FORMAT  ( 1H1  )
  419 FORMAT  (  " ********************************************************"00
     1  "**********",  / ,  " *", T62,  "*",  / ,  " *", T62,  "*",  / 0
     2   ,  " *", T25,  "SAMPLE "  "SIZE = ", I3, T62,  "*",  / ,  " *",00
     3   T62,  "*",  / ,  " *", T22,  "MONTE CARLO "  "SIZE = ", I3,    00
     4   T62,  "*",  / ,  " *", T62,  "*",  / ,  " *", T62,  "*",  / ,  00
     5   " ********************************************************"
     6   "*",  /  /  /  /  /  /  /  )
  429 FORMAT  ( 5(1X,  "COMPONENT ", I1,  / , 6X,  "K = ", F4.2,  / ,  00
     1   6X,  "THETA = ", F5.0,  / , 6X,  "C = ", F2.0,  / , 6X,
     2   "RELIABILITY =", F7.5,  /  / ) )
  439 FORMAT  ( 15(1X, F7.5) )
  449 FORMAT  (/," **********",/," MC=",I4,4X,"J =",I2,4X,"NSTART =",I2
     1                        ,/, " KHAT=" E13.6, 5X "THAT=" E13.6 /00
     2   " ITERATIONS=" I3, 5X "IER=" I3 /  " **********"  )          00
  459 FORMAT  (  /  " ******************" /  /  " DID NOT CONVERGE IN"0
     1   " 8 ATTEMPTS  WITH CIFFERENT STARTING KHATS" /  " THEREFORE GO"00
     2   "ING ON TO THE NEXT   COMPONENT" /  /  " ********************" 0
     3   )
  469 FORMAT ( / / , " THERE WERE", I7, " HIGH TRUNCATIONS OUT OF"
     1  , I8, " RELIABILITY DEVIATES", / / )
```

```
  479  FORMAT  (  /  " ***** SYSTEM 1 *****" /  " (3 COMPONENTS IN SERI"00
     1   "ES)"  )
  489  FORMAT  (  / ,  " TRUE SYSTEM RELIABILITY =", F7.5,  /
     1   " THE 99 PERCENT CONFIDENCE INTERVAL COVERED ",F6.4,
     2   " OF THE RUNS",/,
     3   " THE 95 PERCENT CONFIDENCE INTERVAL COVERED ", F6.4,
     4   " OF THE RUNS",/," THE 90 PERCENT CONFIDENCE INTERVAL COVERED "
     5 , F6.4," OF THE RUNS",/," THE 80 PERCENT CONFIDENCE INTERVAL"
     6   " COVERED ",F6.4," OF THE RUNS",/," THE 70 PERCENT CONFIDENCE "
     7   "INTERVAL COVERED ",F6.4," OF THE RUNS",/," THE 60 PERCENT "
     8   "CONFIDENCE INTERVAL COVERED ",F6.4," OF THE RUNS",/," THE",
     9   " 50 PERCENT CONFIDENCE INTERVAL COVERED ",F6.4," OF THE RUNS",
     1   ///)
  499  FORMAT  (  /  " ***** SYSTEM 2 *****" /  " (1 COMPONENT IN SERIE"0
     1   "S WITH 2 "  "IN PARALLEL)"  )
  509  FORMAT  (  /  " ***** SYSTEM 3 *****" /  " (3 COMPONENTS IN PARA"00
     1   "LLEL)"  )
  519  FORMAT  (  /  " ***** SYSTEM 4 *****" /  " (A 5-COMPONENT COMPLE"00
     1   "X NETWORK)"  )
  529  FORMAT  ( 5(1X, F10.8, 5X) )                                      00
C
       END
```

```
      FUNCTION CMPREL (TIME, K, THETA, C)
      REAL       K
      IF(THETA.EQ.TIME) CMPREL=.36787944
      IF(THETA.EQ.TIME) GO TO 9
      IF(THETA.LT.10E9 .AND.THETA .GE.10E-7) GO TO 6
      IF(THETA.GE.10E9) CMPREL=1.0
      IF(THETA.GE.0.0 .AND.THETA.LT.10E-7) CMPREL=0.0
      IF(THETA.LT.0.0) PRINT 5
    5 FORMAT(1H ,17HTHETA IS NEGATIVE)
      IF(THETA.LT.0.0) CMPREL=0.0
      RETURN
    6 ARGUM=-((TIME-C)/THETA)**K
      IF(ARGUM.GT.-20.0 .AND.ARGUM.LT.0.0) GO TO 8
      IF(ARGUM.GE.0.0) CMPREL=1.0
      IF(ARGUM.LE.-20.0) CMPREL=0.0
      RETURN
    8 CMPREL=EXP(ARGUM)
    9 RETURN
      END
```

```
      SUBROUTINE  REL1 (NMC, R, RS, IDIM)
C  REL1 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN SERIES
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   =1, NMC
   10   RS(I)     =R(I, 1) * R(I,2) * R(I,3)
      RETURN
      END
```

```
      SUBROUTINE  REL2 (NMC, R, RS, IDIM)
C  REL2 DETERMINES THE SYSTEM RELIABILITY OF 1 COMPONENT IN
C  SERIES WITH 2 IN PARALLEL
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   =1, NMC
10    RS(I)       =R(I, 1) * (1. - (1. -R(I, 2)) * (1. -R(I, 3)))
      RETURN
      END
```

```
      SUBROUTINE  REL3 (NMC, R, RS, IDIM)
C  REL3 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN PARALLEL
      DIMENSION   R (IDIM,5)  RS (IDIM)
        DO 10 I   =1, NMC
10    RS(I)       =1. - (1. -R(I,1)) * (1. -R(I, 2)) * (1. -R(I,
     1    3))
      RETURN
      END
```

```
      SUBROUTINE  REL4 (NMC, R, RS, IDIM)
C  REL4 DETERMINES THE SYSTEM RELIABILITY OF A 5 COMPONENT
C  COMPLEX NETWORK
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   =1, NMC
        RS(I)     =R(I, 1) * (1.-(1.-R(I, 2)) * (1.-R(I,5) * (1.-
     1    (1. - R(I,3)) * (1. - R(I,4)))))
10    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PARES(N,M,C,THETA,EK,MR,PTH,PEK,T)
C     INPUT
C     N=SAMPLE SIZE (BEFORE CENSORING),N=100 OR LESS AS
C           DIMENSIONED
C     SS1=0 IF SCALE PARAMETER THETA IS KNOWN
C     SS1=1 IF SCALE PARAMETER THETA IS TO BE ESTIMATED
C     SS2=0 IF SHAPE PARAMETER K IS KNOWN
C     SS2=1 IF SHAPE PARAMETER K IS TO BE ESTIMATED
C     SS3=0 IF LOCATION PARAMETER C IS KNOWN
C     SS3=1 IF LOCATION PARAMETER C IS TO BE ESTIMATED
C     T(I)=I-TH ORDER STATISTIC OF SAMPLE (I=1,N)
C     M=NUMBER OF OBSERVATIONS REMAINING AFTER CENSORING N-M
C           FROM ABOVE
C     C(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF C
C     THETA(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF THETA
C     EK(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF K
C     MR=NUMBER OF OBSERVATIONS CENSORED FROM BELOW
C     OUTPUT
C     N,SS1,SS2,SS3,M,C(1),THETA(1),EK(1),MR
C           --SAME AS FOR INPUT
C     C(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF C
C     THETA(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF THETA
C     EK(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF K
C       (MAXIMUM VALUE OF J AS PRESENTLY DIMENSIONED IS 500)
C     EL=NATURAL LOG. OF LIKELIHOOD FOR C(J),THETA(J),EK(J)
      DIMENSION T(500),C(550),THETA(550),EK(550),X(56),Y(55)
      SS1=1.
      SS2=1.
      SS3=0.
      IF(N) 66,66,104
  104 EN=N
      IF(M) 66,66,110
  110 EM=M
   31 ELNM=0.
      EMR=MR
      MRP=MR+1
   33 NM=N-M+1
      DO 34 I=NM,N
      EI=I
   34 ELNM=ELNM+ALOG(EI)
      IF (MR) 66,35,74
   74 DO 75 I=1,MR
      EI=I
   75 ELNM=ELNM-ALOG(EI)
   35 DO 30 J=1,550
      IF (J-1) 66,25,37
   37 JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
    6 SK=SK+(T(I)-C(JJ))**EK(JJ)
      IF (SS1) 7,7,8
    7 THETA(J)=THETA(JJ)
      GO TO 9
```

```
    8   IF (MR) 66,19,20
   19   THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)
  C**(1./EK(JJ))
        GO TO 9
   20   X(1)=THETA(JJ)
        LS=0
        DO 21 L=1,55
        LL=L-1
        LP=L+1
        X(LP)=X(L)
        ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
        Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)
  C+EK(JJ)*(EN-EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.
  C)-EMR*EK(JJ)*ZRK*EXP(-ZRK)/(X(L)*(1.-EXP(-ZRK)))
        IF (Y(L)) 53,73,54
   53   LS=LS-1
        IF (LS+L) 58,55,58
   54   LS=LS+1
        IF (LS-L) 58,56,58
   55   X(LP)=.5*X(L)
        GO TO 61
   56   X(LP)=1.5*X(L)
        GO TO 61
   58   IF (Y(L)*Y(LL)) 60,73,59
   59   LL=LL-1
        GO TO 58
   60   X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
   61   IF (ABS(X(LP)-X(L))-1.E-4) 73,73,21
   21   CONTINUE
   73   THETA(J)=X(LP)
    9   EK(J)=EK(JJ)
   10   IF (SS2) 12,12,11
   11   DO 17 I=MRP,M
   17   SL=SL+ALOG(T(I)-C(JJ))
        X(1)=EK(J)
        LS=0
        DO 51 L=1,55
        SLK=0.
        DO 18 I=MRP,M
   18   SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))
  C**X(L)
        LL=L-1
        LP=L+1
        X(LP)=X(L)
        ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
        Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)
  C**X(L)+(EN-EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)
  C-C(JJ))**X(L)/THETA(J)**X(L)+EMR*ZRK*(ALOG(ZRK)/X(L))
  C*EXP(-ZRK)/(1.-EXP(-ZRK))
        IF (Y(L)) 43,52,44
   43   LS=LS-1
        IF (LS+L) 47,45,47
   44   LS=LS+1
        IF (LS-L) 47,46,47
   45   X(LP)=.5*X(L)
        GO TO 50
   46   X(LP)=1.5*X(L)
```

```
      GO TO 50
47    IF (Y(L)*Y(LL)) 49,52,48
48    LL=LL-1
      GO TO 47
49    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
50    IF (ABS(X(LP)-X(L))-1.E-4) 52,52,51
51    CONTINUE
52    EK(J)=X(LP)
12    C(J)=C(JJ)
62    IF (SS3) 25,25,14
14    IF (1.-EK(J)) 16,78,78
78    IF (SS1+SS2) 57,57,16
16    X(1)=C(J)
      LS=0
      DO 23 L=1,55
      SK1=0.
      SR=0.
      DO 15 I=MRP,M
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
15    SR=SR+1./(T(I)-X(L))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))
     C**(EK(J)-1.))/THETA(J)**EK(J)-EMR*EK(J)*ZRK*EXP(-ZRK)
     C/((T(MRP)-X(L))*(1.-EXP(-ZRK)))
      IF (Y(L)) 39,24,40
39    LS=LS-1
      IF (LS+L) 70,41,70
40    LS=LS+1
      IF (LS-L) 70,42,70
41    X(LP)=.5*X(L)
      GO TO 22
42    X(LP)=.5*X(L)+.5*T(1)
      GO TO 22
70    IF (Y(L)*Y(LL)) 72,24,71
71    LL=LL-1
      GO TO 70
72    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
22    IF (ABS(X(LP)-X(L))-1.E-4) 24,24,23
23    CONTINUE
24    C(J)=X(LP)
      GO TO 25
57    C(J)=T(1)
25    IF (MR) 66,38,69
38    DO 63 I=1,M
      IF (C(J)+1.E-4-T(I)) 68,67,67
67    MR=MR+1
63    C(1)=T(1)
68    IF(MR) 66,69,31
69    SK=0.
      SL=0.
      DO 36 I=MRP,M
      SK=SK+(T(I)-C(J))**EK(J)
36    SL=SL+ALOG(T(I)-C(J))
      ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
```

```
      EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+
     C(EK(J)-1.)*SL-(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA
     C(J)**EK(J))+EMR*ALOG(1.-EXP(-ZRK))
  150 IF(J-3) 30,27,27
   27 IF (ABS(C(J)-C(JJ))-1.E-4) 28,28,30
   28 IF (ABS(THETA(J)-THETA(JJ))-1.E-4) 29,29,30
   29 IF(ABS(EK(J)-EK(JJ))-1.E-4)126,126,30
   30 CONTINUE
  126 PTH=THETA(J)
      PEK=EK(J)
      GO TO 140
   66 PRINT 135
  135 FORMAT(1H ,20HALL SAMPLES CENSORED,/)
      PEK=0.
      PTH=0.
  140 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RELG1(NMC,R,RS,IDIM,NUM,NALPHA,NSAM)
C  RELG1 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN SERIES
C  AFTER ADJUSTING FOR PERFECT SYSTEM RELIABILITY
      DIMENSION R(IDIM,5), RS(IDIM), F(5,7)
      DATA F/.0488,.1610,.2531,.3721,.4431,.4820,.4943,.0478,
     U .1584,.2493,.3675,.4382,.4771,.4893,.0474,
     2 .1570,.2475,.3652,.4359,.4747,.4870,.0467,
     3 .1550,.2445,.3615,.4319,.4707,.4830,.0464,
     4 .1542,.2435,.3603,.4307,.4694,.4817/
      RSAM=NSAM
      DO 10 I=1,NMC
      RS(I)=R(I,1)*R(I,2)*R(I,3)
   10 IF(RS(I).EQ.1.0) RS(I)=1.0- F(NUM,NALPHA)/RSAM
      RETURN
      END
```

```
      SUBROUTINE RELG2(NMC,R,RS,IDIM,NUM,NALPHA,NSAM)
C  RELG2 DETERMINES THE SYSTEM RELIABILITY OF ONE COMPONENT IN SERIES
C  WITH TWO IN PARALLEL AFTER ADJUSTING FOR PERFECT SYSTEM RELIABILITY
      DIMENSION R(IDIM,5), RS(IDIM), F(5,7)
      DATA F/.0488,.1610,.2531,.3721,.4431,.4820,.4943,.0478,
     U .1584,.2493,.3675,.4382,.4771,.4893,.0474,
     2 .1570,.2475,.3652,.4359,.4747,.4870,.0467,
     3 .1550,.2445,.3615,.4319,.4707,.4830,.0464,
     4 .1542,.2435,.3603,.4307,.4694,.4817/
      RSAM=NSAM
      DO 10 I=1,NMC
      IF(R(I,1).NE.1.0) GO TO 8
      IF(R(I,2).EQ.1.0.OR.R(I,3).EQ.1.0) GO TO 15
    8 RS(I)=R(I,1)*(1.0-(1.0-R(I,2))*(1.0-R(I,3)))
      GO TO 10
   15 RS(I)=1.0-F(NUM,NALPHA)/RSAM
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RELG3(NMC,R,RS,IDIM,NUM,NALPHA,NSAM)
C  RELG3 DETERMINES THE SYSTEM RELIABILITY OF THREE COMPONENTS IN PARALL
C  AFTER ADJUSTING FOR PERFECT SYSTEM RELIABILITY
      DIMENSION R(IDIM,5), RS(IDIM), F(5,7)
      DATA F/.0488,.1610,.2531,.3721,.4431,.4820,.4943,.0478,
     U .1584,.2493,.3675,.4382,.4771,.4893,.0474,
     2 .1570,.2475,.3652,.4359,.4747,.4870,.0467,
     3 .1550,.2445,.3615,.4319,.4707,.4830,.0464,
     4 .1542,.2435,.3603,.4307,.4694,.4817/
      RSAM=NSAM
      DO 10 I=1,NMC
      IF(R(I,1).EQ.1.0.OR.R(I,2).EQ.1.0.OR.R(I,3).EQ.1.0) GO TO 15
        RS(I)     =1. - (1. -R(I,1)) * (1. -R(I, 2)) * (1. -R(I,
     1    3))
      GO TO 10
   15 RS(I)=1.(-F(NUM,NALPHA)/RSAM
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RELG4(NMC,R,RS,IDIM,NUM,NALPHA,NSAM)
C  RELG4 DETERMINES THE SYSTEM RELIABILITY OF A FIVE COMPONENT SYSTEM
C  AFTER ADJUSTING FOR PERFECT SYSTEM RELIABILITY
      DIMENSION R(IDIM,5), RS(IDIM), F(5,7)
      DATA F/.0488,.1610,.2531,.3721,.4431,.4820,.4943,.0478,
     U .1584,.2493,.3675,.4382,.4771,.4893,.0474,
     2 .1570,.2475,.3652,.4359,.4747,.4870,.0467,
     3 .1550,.2445,.3615,.4319,.4707,.4830,.0464,
     4 .1542,.2435,.3603,.4307,.4694,.4817/
      RSAM=NSAM
      DO 10 I=1,NMC
      IF(R(I,1).NE.1.0) GO TO 8
      IF(R(I,2).EQ.1.0) GO TO 15
      IF(R(I,5).NE.1.0) GO TO 8
      IF(R(I,3).EQ.1.0.OR.R(I,4).EQ.1.0) GO TO 15
    8   RS(I)     =R(I, 1) * (1.-(1.-R(I, 2)) * (1.-R(I,5) * (1.-
     1    (1. - R(I,3)) * (1. - R(I,4)))))
      GO TO 10
   15 RS(I)=1.(-F(NUM,NALPHA)/RSAM
   10 CONTINUE
      RETURN
      END
```

VITA

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Computerized Simulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The purpose of this thesis is to compare the accuracy of two Monte Carlo simulation techniques of finding lower system reliability confidence limits: the bivariate technique and the univariate technique. The actual results compared are the confidence interval coverages of the true system reliability associated with the confidence limits.

The bivariate technique is based upon the assumption that the maximum likelihood estimators of the component shape and scale

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parameters have an asymptotic normal distribution. The univariate technique uses the assumption that the component reliability estimates have a normal distribution. Two variations of the univariate technique are also examined. The first variation assumes that component reliability estimates follow a beta distribution instead of a normal distribution. The second variation replaces all perfect system reliability estimates with new, adjusted reliability values.

The results show that the bivariate technique is the most accurate technique if the true system reliability is believed to be below 0.95 in value. The univariate technique which is adjusted for perfect system reliability estimates is the most accurate technique if the true system reliability is believed to be at least 0.95 in value and if the component data used has a sample size of twenty or less. The use of the beta distribution in place of a normal distribution for component reliability estimates in the univariate technique proved to be more accurate only at higher confidence levels and only more accurate than the original univariate technique.